AIM June 6, 2007

1. G finite group $C_1, \dots, C_k$ conjugacy classes in G

$$\frac{1}{|G|} \# \{ [x_1, y_1] \cdots [x_g, y_g] z_1 \cdots z_k = 1 \}$$
$$z_i \in C_i$$

$$= \sum_{\chi \in \mathrm{Irr}(G)} \left(\frac{|G|}{\chi(1)} \right)^{2g-2} \prod_s \frac{|C_s| \, \chi(C_s)}{\chi(1)}$$

Mass formula Frobenius.

2. $G = Gl_n(\mathbb{F}_q)$

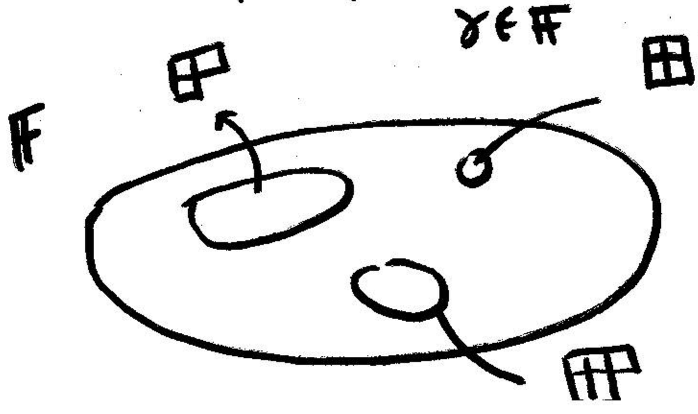
conjugacy classes, irred. characters

$$\Lambda : \mathbb{F} \to \mathcal{P} \quad \text{partitions}$$

$\mathbb{F}$: $\overline{\mathbb{F}}_q^\times$ or its dual

- $\Lambda \circ \mathrm{Frob}_q = \Lambda$
- $|\Lambda| := \sum_{\gamma \in \mathbb{F}} |\Lambda(\gamma)| = n$

E.g



A <u>type</u> τ is the data:

$$(d_1, \lambda^1) \geq (d_2, \lambda^2) \geq \cdots$$

(in some total ordering)

d_i = degre

λ^i = partition

i.e. forget actual eigenvalues.

<u>Note</u>: there are only finitely many types of a given size, independent of base field

$$\frac{|Gl_n(\mathbb{F}_q)|}{\chi_\Lambda(1)} = \mathcal{H}_\tau(q)$$

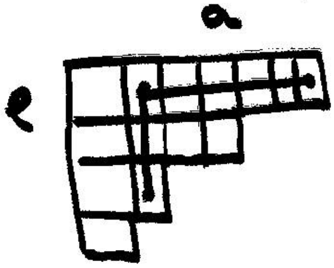
depends only on the type τ of Λ.

Mass formula for $Gl_n(\mathbb{F}_q)$ looks like

$$\sum_\tau \mathcal{H}_\tau(q)^{2g-2} \sum_{\tau(\Lambda)=\tau} \cdots$$

$\mathcal{H}_\tau(q)$ <u>Hook polynomial</u>

$$\mathcal{H}_\lambda(q) = q^* \prod_h (1-q^h)$$



$h = a + l + 1$

In general if $H_\lambda(A)$ is a symmetric function in a set of variables $A = q, t, \underline{x}_1, \underline{x}_2, \ldots, \underline{x}_k$

Extend definition to a type τ by

$$\prod_i H_{\lambda^i}(A^{d_i})$$

raise each variable to the power d_i.

a monomial in A variables

$$a \;\underset{\underset{\longleftarrow}{\text{Log}}}{\overset{\text{Exp}}{\longmapsto}}\; (1-a)^{-1}$$

<u>Prop</u>

$$\text{Log}\Big(\sum_{\lambda\in\mathcal{P}} H_\lambda T^{|\lambda|}\Big)$$

$$= \sum_\tau C^o_\tau H_\tau T^{|\tau|}$$

$$C^o_\tau = \begin{cases} \dfrac{\mu(d)}{d}(-1)^{m_d-1}\dfrac{(m_d-1)!}{\prod_\lambda m_{d,\lambda}!}, & \tau=(d,\lambda^1)\geq(d,\lambda^2)\geq\cdots \\ 0 & \end{cases}$$

Deformation of Hook polynomial

$$\mathcal{H}_\lambda(z,w) := \prod \frac{\left(z^{2a+1} - w^{2l+1}\right)^{2g}}{\left(z^{2a+2} - w^{2l}\right)\left(z^{2a} - w^{2l+2}\right)}$$

$$\Omega_{g,k}(z,w) := \sum_{\lambda \in \mathcal{P}} \prod_{s=1}^{k} \tilde{H}_\lambda(\underline{x}_s; z^2, w^2)\, \mathcal{H}_\lambda(z,w)$$

↑ Macdonald polynomial

$$\langle \mathrm{Log}\, \Omega_{g,k},\ h_{\mu^1}(\underline{x}_1) \cdots h_{\mu^k}(\underline{x}_k) \rangle =: \frac{\mathbb{H}_\mu(z,w)}{(z^2-1)(1-w^2)}$$

<u>CONJ</u> MH polynomial of M_μ is

$$(t\sqrt{q})^{\dim M_\mu}\, \mathbb{H}_\mu\left(-\sqrt{q}, 1/t\sqrt{q}\right)$$

<u>$g=0,\ k=2$</u> orthogonality of $\tilde{H}_\lambda$

$$\Leftrightarrow \quad \mathbb{H}_\mu(q,t) = \begin{cases} 1 & \mu = (1),(1) \\ 0 & \text{otherw.} \end{cases}$$

$\Rightarrow$ conjecture since

$$M_\mu = \begin{cases} \text{point} & \ldots \\ \phi & \ldots \end{cases}$$

<u>PROVE</u> Two specializations

- $z = 1/\sqrt{q}$, $w = \sqrt{q}$ E-polynomial of M_μ (μ indivisible)
- $z = 0$, $w = \sqrt{q}$ E-polynomial of Q_μ quiver variety