G. Lehrer II June 5, 2007

Degree of representation?

$$HC_{L_1 \times L_2}^{G_n}(\chi_1 \otimes \boxtimes \chi_2) =: \chi_1 \circ \chi_2$$

$\bigoplus_{n \geq 0} \Gamma(G_n)$ **Hopf Algebra**

$\chi \in Irr(G_n) \longleftrightarrow \lambda : \hat{\mathbb{F}} \to \mathcal{P}$

$\langle \psi_1 \rangle, \ldots, \langle \psi_r \rangle$ $\quad J^{\langle \psi_1 \rangle}(\lambda_1) \circ \cdots \circ J^{\langle \psi_r \rangle}(\lambda_r)$

$\lambda_1, \ldots, \lambda_r$

$$\deg(HC_L^{G_n}(\alpha)) = \alpha(1)[G_n : P], \quad L \subseteq P$$

geometrically $\hookrightarrow U \to G/L$ (affine) $\downarrow$ $G/P \leftarrow$ complete

Reduced to compute:

$d = \deg \psi, \quad |\lambda| = n/d$

Recall

$$R_{T_\lambda}(\psi) = HC(J_{\lambda_1}^{\langle \psi_1 \rangle} \otimes \cdots)$$

$$J^{\langle \psi \rangle}(\lambda) = \text{l.c. of } R_{T_\mu}(\psi_\mu)\text{'s}$$

Example

$T = (\diagdown)$ $\quad HC_T^G(1) = 1_B^G = \bigoplus_{|\lambda| = n} \chi^\lambda(1) u_\lambda$ $\leftarrow$ principal series

degree $= \prod^{n-1} (q^i - 1)$

$$u_\lambda = J^{\langle 1 \rangle}(\lambda) = \frac{1}{|W|} \sum_{w \in W} \chi^\lambda(w) R_{T_w}^{(1)}$$

$$R_{T_\lambda}(\psi) \cdot St_{G_n} = \mathrm{Ind}_{T_\lambda}^{G}(\psi)$$

" Deligne - Lusztig describe how to divide by zero " St_{G_n} is 0 on any element except semisimple

$$\deg R_{T_w}^{(1)} = \underset{\varepsilon(w)}{\wedge} \frac{|G^F|}{|T^F|} = \frac{\prod_{i=1}^{n} (q^i - 1)}{\prod (q^c - 1)}$$

c cycle length

St_{G_n} dim $q^{\frac{1}{2} n(n-1)}$

$\overline{G}_n := Gl_n(\overline{\mathbb{F}}_q)$, $\overline{B}_n = \begin{pmatrix} 0 & * \end{pmatrix}$

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$\overline{G}_n / \overline{B}_n = \mathcal{B}$ flag variety (complete variety)

THM Let $u \in G$ be unipotent there is an action of W on $H^*(\mathcal{B}_u)$ where

$\mathcal{B}_u := \{ B' \mid u B' \}$ &

(B' ↑ Borel)

$$R_{T_w}(1)(u) = \sum_i \mathrm{trace}(w, H^{2i}(\mathcal{B}_u)) q^i$$

$$J_n^{\langle\psi\rangle}(1) = (q-1)\cdots(q^{n-1}-1)$$

$$\Vert$$

$$R_{T_{cox}}^{\langle\psi\rangle}(1) = \sum_i \operatorname{trace}(cox, H^{2i}(\mathcal{B}))$$

↑ coinvariant algebra of W

e.g. <u>$n=2$</u> $\quad 1-q$

<u>$n=3$</u> $\quad S_3 \quad 1, \rho, \varepsilon$ irred. char.

S_0	S_1	S_2	S_3
1	ρ	ρ	ε

values on (123)

1	-1	-1	1

$$1-q-q^2+q^3$$

"Find" $\quad \sum_i \langle \chi, H^{2i}(\mathcal{B}_u)\rangle_W \, q^i$

<u>E.g.</u>

$u = u_J$ regular in $L = L_J \quad J \subseteq \Pi$

$\rho_k = \Lambda^k$ reflection repn

<u>Lehrer-Shoji</u>

$X_J^0 = \left(\bigcap_{\alpha\in J} H_\alpha\right) \setminus \bigcup_{H \not\supseteq \cap H_\alpha} H$ / $f_{\rho_k}(q) = \sigma_k(q^{b_1(J)}, q^{b_2(J)}, \ldots, q^{b_{n-|J|}(J)})$, where $n-|J|$

$\uparrow$ elementary symmetric fct $\quad P_{X_J^0} = \prod_{i=1} (1+b_i(J)t$