

G. Lehrer II June 5, 2007
Degree of representation?

$$HC_{L_1 \times L_2}^{G_n}(\chi_1 \otimes \chi_2) =: \chi_1 \circ \chi_2$$

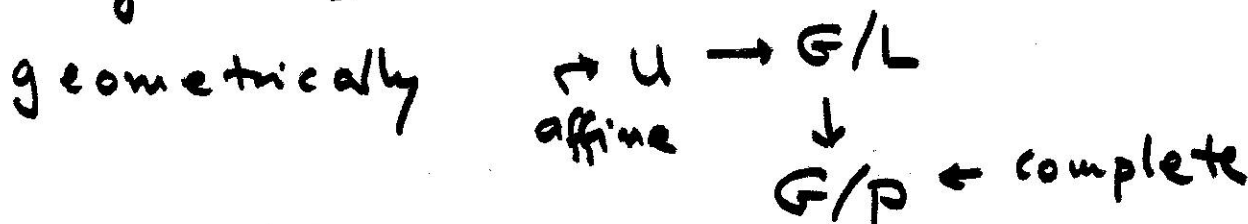
$\oplus_{n \geq 0} \Gamma(G_n)$ Hopf Algebra

$$\chi \in \text{Irr}(G_n) \leftrightarrow \lambda: \hat{\mathbb{F}} \rightarrow \mathcal{O}$$

$$\langle \psi_1 \rangle, \dots, \langle \psi_r \rangle \quad J_{(\lambda_1)}^{\langle \psi_1 \rangle} \circ \dots \circ J_{(\lambda_r)}^{\langle \psi_r \rangle}$$

$\lambda_1, \dots, \lambda_r$

$$\text{deg}(HC_L^{G_n}(\alpha)) = \alpha(1) [G_n : P], \quad L \subseteq P$$



Reduced to compute:

$$d = \text{deg } \psi, \quad |\lambda| = n/d$$

Recall

$$R_{T_\lambda}(\psi) = HC_{\lambda}(\bigoplus_{\lambda_i} \langle \psi_i \rangle \dots)$$

$$J_{\langle \psi \rangle}(\lambda) = \text{l.c. of } R_{T_\mu}(\psi_\mu) \text{'s}$$

Example

$$T = (\setminus)$$

$$HC_T^G(1) = \frac{1}{B} = \bigoplus_{|\lambda|=n} x^{d(\lambda)} u_\lambda$$

degree = $\prod_{i=1}^{n-1} (q^i - 1)$ ↑
principal series

$$u_\lambda = J^{<1>}(\lambda) = \frac{1}{|W|} \sum_{w \in W} \chi^\lambda(w) R_{T_w}^{(1)} \quad (2)$$

$$R_{T_\lambda}(\gamma) \cdot St_{G_m} = \text{Ind}_{T_\lambda}^G(\gamma)$$

"Deligne-Lusztig describe how to divide by zero" St_{G_m} is 0 on any element except semisimple

$$\deg R_{T_w}^{(1)} = \frac{|G^F|}{|T^F|} = \frac{\prod_{i=1}^n (q^i - 1)}{\prod (q^c - 1)}$$

c cycle length

$$St_{G_m} \quad \dim q^{\frac{1}{2}n(n-1)}$$

Springer $\bar{G}_m := GL_n(\bar{\mathbb{F}}_q)$, $\bar{B}_m = \begin{pmatrix} \circ & * \\ & \end{pmatrix}$

$\bar{G}_m / \bar{B}_m = B$ flag variety (complete variety)

THM Let $u \in G$ be unipotent there is an action of w on $H^*(Bu)$ where

$$B_u := \{ B' \mid u \in B' \} \quad \&$$

$$R_{T_w}^{(1)}(u) = \sum_i \text{trace}(w, H^{2i}(Bu)) q^i$$

$$J_n^{<\psi>}(1) = (q-1) \dots (q^{n-1}-1)$$

$$R_{T_{cox}}^{<\psi>}(1) = \sum_i \text{trace}(cox, H^{2i}(B))$$

↑
coinvariant algebra
of W

e.g. n=2 1-q

n=3 S₃ 1, ρ, ε irred. char.

S ₀	S ₁	S ₂	S ₃
1	ρ	ρ	ε

values on (123)

1	-1	-1	1
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$$1 - q - q^2 + q^3$$

"Find" $\sum_i \langle \alpha, H^{2i}(B_u) \rangle_W q^i$

E.g. u = u_J regular in L = L_J J ⊆ Π

ρ_k = 1^k reflection rep_u

Lehrer-Skoffi

$$X_J^0 = \left(\prod_{\alpha \in J} H_\alpha \right) \cup H / f_{\rho_k}(q) = \sigma_k(q^{b_1(J)}, q^{b_2(J)}, \dots, q^{b_{n-1}(J)})$$

↑
elementary symmetric

where n = |J|
P_{X_J⁰} = ∏_{i=1}ⁿ (1 + b_i(J)t)