COMPACT MODULI SPACES AND BIRATIONAL GEOMETRY

organized by
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Workshop Summary

This report summarizes the scientific activities of the meeting, as well as subsequent developments.

For the most part, morning sessions were devoted to presentations outlining core results and open problems in the field. These were to be accessible to all participants, including graduate students, recent PhD's, and nonspecialists. The goal was to provide context and motivation for the research questions in the area:

(1) János Kollár (Princeton): Compact moduli spaces: history and current problems
(2) Paul Hacking (Yale): Compact moduli of plane curves
(3) Eckart Viehweg (Universität Duisburg-Essen): Compactification and constructions of moduli
(4) Valery Alexeev (Georgia): Boundedness of log pairs
(5) Ian Morrison (Fordham): Stability of Hilbert Points
(6) Sean Keel (Texas): Log canonical compactifications
(7) Nick Shepherd-Barron (Cambridge): An example of toroidal compactifications

The meeting opened with Kollár's lecture, which gave a broad survey of the field. Viehweg discussed techniques for proving quasi-projectivity of moduli spaces of higher-dimensional varieties. Alexeev offered a survey of methods for proving moduli spaces are of finite type. Morrison gave a history of applications of Geometric invariant theory to moduli problems, and mentioned some exciting new connections with differential geometry. Hacking described his application of Artin's criterion to construct moduli stacks of stable varieties, as well as his example of plane curves. Keel discussed how the log minimal model program might be applied to moduli spaces themselves, with particular emphasis on hyperplane arrangements. Shepherd-Barron sketched out a beautiful example where a toroidal compactification can be given a modular geometric interpretation.

There were a number of problem sessions, conducted by Karen Smith of the University of Michigan. These included formal statements of open problems, as well as basic questions of interest to the participants.

There were also a number of scheduled short presentations of research results relevant to the themes of the conference:

(1) Jenia Tevelev (Texas): Tropical Compactifications
(2) Samuel Grushevsky (Princeton): A_6 is of general type
(3) Yongnam Lee (Sogang University): On connectedness of the space of maps
(4) Martin Olsson (Institute for Advance Study): Log geometry and moduli of K3 surfaces
(5) Valery Alexeev (Georgia): Stable pairs with group action and their moduli

(6) Gavril Farkas (Texas): Syzygies and the effective cone of the moduli space of curves
(7) Paul Hacking: Compact moduli of hyperplane arrangements
(8) Jenia Tevelev: Membranes and crepant resolutions of visible contours
(9) Angela Gibney (Pennsylvania): A generalization of the moduli space of pointed rational curves
(10) Eyal Markman (Massachussetts): The birational Torelli problem for hyperkähler varieties
(11) Ravi Vakil (Stanford): Murphy's Law in algebraic geometry: Badly behaved deformation spaces
(12) Yuri Prokhorov (Moscow State University): On normal degenerations of Del Pezzo surfaces

There were also a number of informal presentations on very new research results:

(1) James McKernan (UCSB) presented his new results on boundedness for $\mathbb{Q}$-Fano varieties.
(2) Brendan Hassett (Rice) discussed recent work with Dan Abramovich, using quotient stack presentations of stable varieties to construct moduli spaces.
(3) Max Lieblich (Brown) presented his results on the period-index problem for the Brauer group of the function field of a surface, which uses moduli spaces of twisted sheaves over the surface.
(4) János Kollár sketched some new counterexamples, which are discussed in more detail below.

Two new results arose during the week of the conference: the counterexamples of Kollár. First, he gave an example of a moduli space of polarized varieties which is not quasiprojective. This seems to contradict recent results of Schumacher and Tsuji published in the *Annals of Mathematics*. A preprint describing this example is available:

J. Kollár, Non–quasi–projective moduli spaces, math.AG/0501294

The second counterexample shows that different formulations of moduli functors of stable varieties are inequivalent in positive characteristic. One formulation, due to Viehweg, stipulates that *some* fixed reflexive power of the dualizing sheaf $\omega_\pi^{[N]}$ is locally free. The other formulation, due to Kollár, stipulates that *every* reflexive power of the dualizing sheaf commutes with base extension. In characteristic p, when N is replaced by Np^e with increasing e, Viehweg's approach can give an infinite sequence of inequivalent functors.

Several new initatives came out of the conference. For over a decade, there have been plans for a book on moduli of stable surfaces. However, this has been stalled by a number of technical issues. One was the ambiguity about the right choice of moduli functor, which is resolved by the counterexample sketched above. Another was local closedness of the moduli functor, which is addressed in Hacking's thesis and subsequently in the work of Abramovich and Hassett alluded to above. With these issues resolved, Kollár, Kovács, and Hassett plan a book giving a comprehensive account of this story.

Another initiative is that Hassett and Kovács (along with William Fulton and Vakil) have submitted a preproposal for a year-long program at MSRI on algebraic geometry. The topics addressed in this workshop will play a substantial rôle in this project, if it is approved.

Finally, the organizers feel that the environment at the present AIM conference site is quite conducive to positive scientific interaction. While the physical space is unconventional (we all make jokes about working out of the back of a warehouse), this does not create any serious problems. Indeed, the fact that the talks were held in the large hall where most of the other activities took also place gave an ideal background to faciliate one of the foremost purposes of AIM: joint work involving most of the participants. Discussions were easily extended to include as many people as were interested in them, sometimes turning into ad hoc presentations by one of the participants.

It should also be noted that if AIM were to move to a location isolated from Palo Alto, there might be substantial drawbacks, e.g., access to restaurants and other cultural resources would be less convenient.