Legendrian and transverse knots

organized by
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Workshop Summary

The list of participants of this workshop included almost all people active in the title area (among the most regrettable exceptions, we can mention John Etnyre, who had to cancel his planned participation because of a home emergency, and Peter Pushkar, who was not able to resolve visa problems; still, Peter mailed to the organizers a description of his planned talk, which was distributed and thoroughly discussed in the last days of the workshop). Also, there were some young mathematicians who, certainly, learned a lot during this week of an extremely intense work.

As it is common for the AIM workshops, every day before lunch, there were a few talks addressed to all the workshop participants.

On the first day of the workshop, both talks were delivered by Josh Sabloff. The common title of the two talks was "An introduction to invariants of Legendrian knots". Besides classical invariants (topological type, the Thurston-Bennequin number, the Maslov number), the speaker clearly explained the definition of Chekanov-Eliashberg DGA and other versions of contact homology, normal rulings, generating families of functions. The speaker presented a fairly complete account of the relations between these invariants (results of Bennequin, Tabachnikov, Fuchs, Chmutov, Goryunov, Etnyre, Honda, Epstein, Pushkar, Eliashberg, Chekanov, Ng, Sabloff, Ishkhanov, Rutherford; it is worth mentioning that of these 15 authors, 10 were among the participants of the workshop).

The relations between generating families of functions, normal rulings and augmentations in the Chekanov-Eliashberg DGA were discussed in two more talks: of Dan Rutherford and of Michael (Brad) Henry. Also, these relations were the subject of the discussion section, which was, probably, the most populated and the most long living (it worked during all the five days of the workshop). The talk of Dan Rutherford on Tuesday contained an account on a recent work by Rutherford and Fuchs on relations between generating families of functions and the homology of the Chekanov-Eliashberg DGA linearized by an augmentation. The talk of Michael (Brad) Henry concerned the multidimensional case of the problem; although this subject is new, there are some exciting results and conjectures.

The talk of Lenny Ng on Tuesday had an ambitious title "Legendrian symplectic field theory". The goal of the author is to create a rational symplectic field theory for Legendrian knots that would contain contact homology and more. The hope of the author (which has not materialized yet) is that the invariants arising will be non-trivial for stabilized Legendrian knots; this will make it possible to apply them to transverse and even topological knots. The construction proposed is similar to (but more sophisticated than) the most classical construction of the Chekanov-Eliashberg DGA; the most important difference is that the algebra considered has two, not one, generator associated to every crossing of the diagram

of the knot. Also, the coefficients are integers, not residues mod 2. A differential is defined and it requires considerable efforts to make the square of the differential zero.

It must be mentioned here that Lenny delivered one more talk, a shorter one, in the last day of the workshop. The title "Knot contact homology and string theory" speaks for itself. One can hope that the relations between contact homology and string topology will be discussed more closely in the future workshops.

On Tuesday morning, in addition to the above mentioned talk of Henry, there was also a talk by Hansjorg Geiges that discussed contact versions of some classical topological constructions. In particular, it was discussed how every closed, connected contact manifold can be obtained by +1 or -1 surgery along a Legendrian link. In addition, contact versions of moves from Kirby calculus were discussed.

On Wednesday morning, there were three interrelated talks by Vera Vertesi, David (Shea) Vela-Vick, and Ko Honda. The talks were devoted to invariants stemming from different versions of Floer homology: combinatorial knot Floer homology (aka Heegard Floer homology) and sutured Floer homology. This subject was also discussed in a two days long afternoon discussion section.

A joint talk was delivered on Friday by Lisa Traynor and Josh Sabloff. The talk established a connection between contact and symplectic geometries. The authors consider a flat at the infinity Lagrangian manifold in the 4-dimensional space and the family of slices by a family of hyperplanes. There are some results concerning possible slices and possible variations of slices within the family, and there are exciting unsolved problems.

Also on Friday, Yasha Eliashberg delivered a talk "Stein fillable contact manifolds". The speaker presented a different view of contact geometry with main ideas discussed on the workshop presented from a new point of view: contact homology, generating families, augmentations etc. arise in the context of a contact manifolds, and their relations with Legendrian knots are established by means of a contact surgery.

All the participants of the workshop worked hard in the afternoon discussion sections. The first afternoon was spent generating a list of problems that were conveyed orally to Paul Melvin who then formalized them into a written list on the board; Dan Rutherford wrote up the list that was distributed to all workshop participants. This problem generating session formed the basis of the afternoon working groups for the other days. The section "Generating families, normal rulings, and augmentations" worked during all the four days, and even on Friday, when no meeting was scheduled, people gathered in the discussion room and left only to catch their flights at the airport. The participants included Lisa Traynor, Josh Sabloff, Brad Henry, Dan Rutherford, Paul Melvin, Dmitry Fuchs, Sergei Chmutov, Victor Goryunov with some other people (Sergei Tabachnikov, Lenny Ng) occasionally visiting. It goes without saying that the younger participants (Josh, Brad, and Dan) were the most active. Although the existences of normal rulings, generating families, and augmentations are mutually equivalent, there is no established relations between the classes of these three objects. Some people in the room had ready counter-examples to fine conjectures presented by some other people. In the end it was generally accepted that the notion of a normal ruling must be modified; the modified notion already exists, it has been developed independently by Pushkar and Ng-Sabloff under the name of a "decorated normal ruling". No immediate objections were found to a possibility of a bijective correspondence between (classes of) decorated normal rulings, generating families, and augmentations. The work is to be continued.

Other discussion sections that worked for a day or two concerned virtual Legendrian knot (the common opinion was that this is a very interesting but hard project: it is not even clear what the main definition should be); Legendrian unknotting number (some easy estimates relating the Legendrian unknotting number and the topological unknotting number were found), tabulation of Legendrian knots and Heegard Floer homology.

In our opinion, the workshop was highly successful: it has brought together most of the workers in the field and significantly energized this active area of research.