AIM Workshop on Configuration Spaces of Linkages
Open Problems

Notes by Elissa Ross

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1. (Farber) Configuration spaces of closed polygons in $\mathbb{R}^3$. Let $\ell = (\ell_1, \ldots, \ell_n)$ be the length vector defining a closed polygon.

   The configuration space is $M_\ell \backslash \Sigma = \sqcup U_i$, where U_i are the connected components, and Σ is the set of configurations with self-intersections.

   Questions: Are the U_i contractible? What can we say about their topology?

2. (Streinu) Same as the previous question, but with an open arm in $\mathbb{R}^3$, and $\ell_1 = \ell_2 = \cdots = \ell_n$.

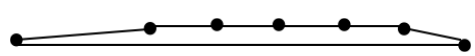
   Question: Is the space of non-intersecting configurations connected?

3. (Farber/Panina) Consider a closed polygon in $\mathbb{R}^3$. According to Klyatchko, M_ℓ has a symplectic form for ω, and therefore for volume.

$$Vol\Big(\sqcup U_i\Big)\Big/Vol(M_\ell),$$

   where i corresponds to an unknot.

   (Note if $\ell_1 + \ell_2 + \cdots + \ell_{n-1} = \ell_n$, this is always an unknot and therefore the proportion above is exactly 1.(see below))



   Question: try to understand this proportion.

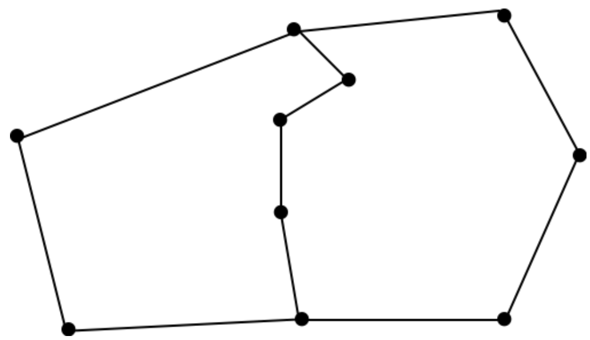
4. (Holmes-Cerfon) Consider the configuration space of planar n-gons such that

$$L_i - \epsilon \leq \ell_i \leq L_i + \epsilon.$$

   $M_{L,\epsilon}$ is a manifold with boundary.

   Question: Understand its topology and volume.

5. (Sitharam) Consider two polygons that share a "chain" (see below), or graphs of tree-width 2.



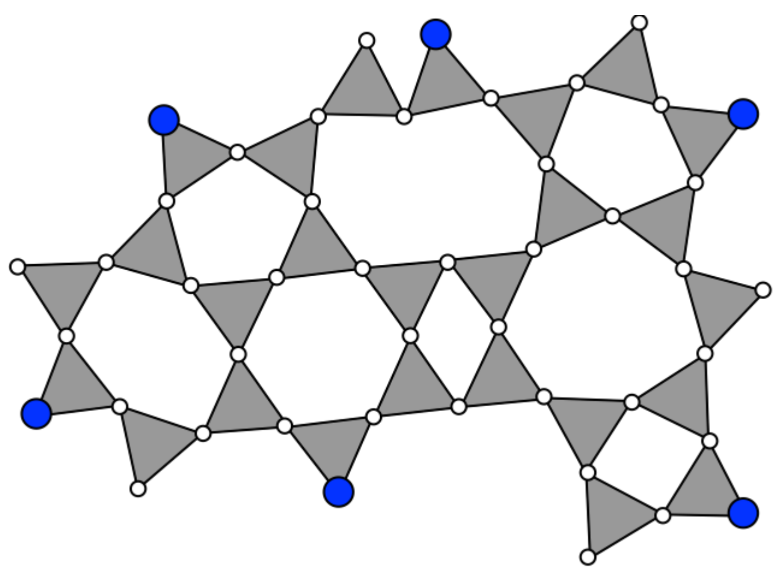
Question: Apply Morse Theory à la Farber to derive the Betti numbers:

a) Chambers

b) Homologies

6. (Farber) Question: Find asymptotic behaviour of C_n (the number of orbits of chambers in the case of polygonal linkages).

7. (Thorpe) Consider a network of corner-sharing triangles in the plane, with holes of size $5, 6, 7, 8, 9$ (and an average hole size of 6). If this is an infinite network, it is isostatic.



Now take a large finite piece of the framework. Experimentally, if we pin every other triangle boundary vertex (vertex of degree two) and run the pebble game, we get an isostatic network.

Question: prove that this approach works in general and find other distributions of pins that also work.

a) generic

b) equilateral triangles

Question (generalization): What about the class of graphs that have no proper rigid subgraphs, but that have more than two bodies at a pin? The underlying body-pin graph is no longer 3-regular.

8. (Hempel) Consider a simplicial polyhedra with fixed combinatorics in $\mathbb{R}^3$.

   Question: characterize those collections of dihedral angles and face angles that can be realized by a polyhedron with these combinatorics. Conjecture: dimension $= E - 1$, where E is the number of edges. Needs more clarification.

9. (St. John) Consider a multi-robot formation that is a generically minimally rigid framework G, with diameter $D = \max_{(p_i,p_j)} ||p_i - p_j||$. We remove an edge and obtain $\bar{G}$ that is flexible. Let $d = \min$ diameter(configuration space of $\bar{G}$).

   Question: Understand the relationship between D and d, and find algorithms to detect what edge to delete for maximal change in diameter. More precisely, find find bar e and "positioning" q such that the diameter of $(G \backslash e, q)$ is minimum under the constraints that bar lengths in $G \backslash e$ are maintained.

10. (Theran) Consider a Delaunay triangulation (no vertex is inside the circumcircle of any triangle). Fix a combinatorial type of a triangulation.

    Question: What is the configuration space?

    $d = 2$ it is a ball

    $d = 3$ is it universal?

11. (Owen) Specialization: When is the Galois group of a particular rigid framework a subgroup of the Galois group of the graph?

    When is $\mathcal{G}(G,p) \subseteq \mathcal{G}(G)$ for any p with (G,p) isostatic? (Isostatic is sufficient but not necessary).

    Here p generic means $p = x_1, \ldots, x_n$ are algebraically independent over $\mathbb{Q}$.

12. (Whiteley) Conjecture: Given a symmetric bar-joint framework (G,p), the configuration space of (G,p) (with appropriate parts of the frame of reference fixed), has the symmetry of the most symmetric individual realization in the configuration space.

13. (Schulze) Understand the following question: does the pseudo triangulation algorithm for the Carpenter's Rule give some unfolding that preserves symmetries? (Note that Connelly, Demaine, Rote have non-algorithmic positive solution).

14. (Schulze) Suppose a symmetric framework (linkage) has a 1DOF expansive mechanism. Does the mechanism preserve the symmetry?

15. (Schulze) Under what conditions is a linkage guaranteed to preserve the original symmetry throughout the configuration space?

16. (St. John/Schulze) Persistence theory: Group of connected agents, every agent has out-degree 2 (except for 2 agents, the leader and the co-leader).

    Question: Can symmetry of the configuration be exploited to reduce the computation? What about a body-CAD version?