Complex Monge-Ampère Equation Workshop: Open problems

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September 8, 2016

1. Monge-Ampère masses supported by analytic sets

   (a) Compact Kähler case (e.g., $\mathbb{C}P^n \setminus H$) [S. Dinew]

   - (A good) definition? Possible criteria:
     1. If $u_n \downarrow$ and $v_n \downarrow$, $u_n, v_n \in L^\infty \cap PSH$ and $\lim u_n = \lim v_n$, then $\lim MA(u_n) = \lim MA(v_n)$.
     2. Take u and $u_n = \max(u, -n)$. Try $MA(u) = MA(u_n)$ (if it exists).
   - Solvability of $MA(u) = \mu$, where μ = measure supported on an analytic set (not a point), Green's function, possibly with prescribed singularities.

   (b) Similar questions for the Dirichlet problem on $\Omega \subset\subset \mathbb{C}^n$.

   (c) Suppose $\not\exists$ KE metric on X, with $c_1(X) > 0$: Consider e.g. the continuity method $\psi_t := \phi_t - \sup \phi_t$. Show that a subsequence $\psi_t \to \psi$ whose $MA(\psi)$ is defined and $MA(\psi) = \mu$, μ supported in the multiplier ideal sheaf. (e.g., $X = \mathbb{C}P^2$ with one point blown-up.)

   (d) Real analogues (from the Toric case, for example).

   (e) X Fano, D smooth anti-canonical divisor. Let ω_ϵ satisfy

   $$\mathrm{Ric}(\omega_t) = \epsilon\omega_t + (1-\epsilon)[D].$$

   What is the limit of (subsequence) ω_ϵ? In particular, is its Ricci (in a suitable sense) supported on D? [H. Guenancia]

2. Are there non-product solutions of $(dd^c u)^n = 0$ on M compact (e.g., $\mathbb{C}P^2$), where u is smooth and not necessarily PSH? [Y. Rubinstein]

3. Are there solutions of $(dd^c u)^n = 0$ on $\mathbb{C}^n$, where u is PSH? [W. He]

4. Let Ω be a strongly pseudo-convex domain, $\partial\Omega \in C^{3,1}$. Consider the problem $(dd^c u)^n = f$, $f \geq 0$, $f^{\frac{1}{n}} \in C^{1,1}$ and $u|_{\partial\Omega} = \phi$, $\phi \in C^{3,1}(\partial\Omega)$. Find an analytic (independent from Krylov's) proof of $u \in C^{1,1}(\overline{\Omega})$.

5. Same question as above but with $f^{\frac{1}{n-1}} \in C^{1,1}$ (in this case, not covered by Krylov.) (Special case known: $\Omega = B^n$, $\phi = 0$: yes by Pliś.)

6. Can one construct a counterexample to the maximal rank question from the example of Ross-Witt-Nystrom of solutions of the HCMA without foliation? [M. Paun]

7. Solving $(\pi^*\omega + i\partial\overline{\partial}u)^{n+1} = 0$ on $X \times A$, where A is an annulus, and ω is possibly degenerate. $u|_{X\times\{t=1,e\}} = \phi_0, \phi_1$, ϕ_0 and ϕ_1 ω-psh (with some regularity) and $\int_X \omega^n > 0$. [E. Di Nezza]

8. Find a PDE proof of Kolodziej's L^∞ estimate; find optimal constant for a ball. [Z. Błocki]

9. Find X KE Fano and $u \in \Lambda_1$ such that $\int_X u^3\omega_{KE}^n \neq 0$. [H. Macbeth]

10. $(dd^cu)^n = 1$ on Ω, $\Delta u \in L^{n(n-1)} \implies u \in C^\infty$? [T. Collins]

11. Complex version of Pogorelov's estimate.

12. Let $p : X \to \mathbb{D}$, X Kähler and K_X nef. Study solutions of $\mathrm{Ric}(\omega_\epsilon^t) = -\omega_\epsilon^t - \epsilon\beta_t$ on X_t. [M. Paun]

13. Find an analytic proof of the ACC Conjecture/Theorem. [T. Collins]