Rational curves on algebraic varieties

organized by
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Workshop Summary

The workshop had a great atmosphere and provided a stimulating environment for both existing collaborations and new partnerships.

The mornings were devoted to 'agenda-setting' talks by senior participants:

M	9:00	Brendan Hassett	Overview
		Sándor Kovács	
	9:20	Y. Miyaoka	Introduction to rationally connected varieties
	11:00	J.L. Colliot Thélène	Arithmetic surjectivity and geometric surjectivity
T	9:00	J. Starr	Rationally simply connected varieties and applications
	11:00	H. Esnault	Rationally connected varieties over finite fields
W	9:00	S. Kebekus	Rational curves and foliations
	11:00	Y. Tschinkel	Weak approximation
Th	9:00	J.M. Hwang	Varieties of minimal rational tangents
	11:00	Y. Miyaoka	Characterization of projective space
F	9:00	F. Campana	Special varieties and rationally connected orbifolds
	11:00	J. de Jong	Recent progress on Serre's conjecture

There were also some afternoon lectures on current developments. For example, J. Starr spoke on recent weak approximation results for smooth hypersurfaces $X \subset \mathbb{P}^n$ of degree $d \leq \sqrt{n}$, defined over function fields of complex curves. Y. Prokhorov spoke on joint work with S. Mori classifying conic bundle fibrations. A. Asok spoke on $\mathbb{A}^1$-homotopy theory.

Karen Smith led a lively problem session Monday afternoon, which gave the participants the opportunity to formulate questions. These included both open research problems and general background questions. The working groups were organized around these problems:

(1) cones of curves on rationally connected and Fano varieties;
(2) rational curves on non-proper varieties and the smooth locus of singular Fano varieties;
(3) weak approximation over function fields;
(4) the competing definitions of 'rationally simply connected varieties' developed for various purposes;
(5) arithmetic questions, e.g., rationally connected varieties over function fields of real curves.

The workshop was attended by many young researchers who either have not or only recently entered this area of research. They were successfully integrated into the working groups and participated actively.

Starting Tuesday, the first half-hour of the afternoon session was devoted to short presentations by the working groups, including example/counterexamples related to the questions on the problem list. For example, Ana-Maria Castravet presented counterexamples to an overly-optimistic statement made the previous afternoon.

The first working group contemplated questions about log Fano varieties that were raised at the open problem session. Many questions revolved around the geometry of the cone of effective curves, e.g., how does the cone of curves vary in a family of smooth Fano varieties? This was a popular working group for younger participants as the problems were somewhat easier to comprehend (but not necessarily to solve).

The second working group concentrated on specific questions arising from extending the theory of rational-connectedness to open varieties. For instance, can a non-log-general-type uniruled variety be covered by copies of $\mathbb{A}^1 \setminus 0$? This is known in dimension two due to work of Keel and McKernan, but unfortunately their proof is rather intricate and hinges on a complicated case-by-case analysis. This group attracted some of the younger participants and resulted in lively discussions throughout the week. A few of the participants (including Hassett, Kebekus, Kovács and Tschinkel) have already agreed on a continuation of this effort in a concentrated, small group setting next summer at the Max Planck Institute for Mathematics in Bonn, Germany. This ensures a very long term impact of the current workshop. The plan is to revisit the Keel-McKernan result and obtain a more conceptual proof.

The research questions considered by the working groups 'weak approximation' and 'rationally simply connected varieties' will be pursued by an NSF-funded Focused Research Group led by de Jong, Hassett, Starr, Tschinkel.

The workshop succeeded in encouraging cross-fertilization across different areas:

- A smooth complex variety is *strongly rationally connected* if any two points can be joined by a proper rational curve. Esnault proposed cohomological obstructions to this property.
- Let k be a field of cohomological-dimension two and G/k a simply-connected semi-simple algebraic group; Serre conjectured that $H^1(k, G)$ is trivial. de Jong brought geometric constructions (the space of rational curves on a variety, the space of sections of a fibration) to prove this over the function field of a complex surface in the remaining case $G = E_8$.
- The arithmetically-oriented participants had many discussions with Starr and Hassett about proving weak approximation for rationally-connected fibrations over complex curves.
- The complex-differential viewpoint on varieties of minimal rational tangencies lead to interesting open questions in birational geometry and classical algebraic geometry, e.g., Hwang's problems on the variety of lines in a cubic threefold.