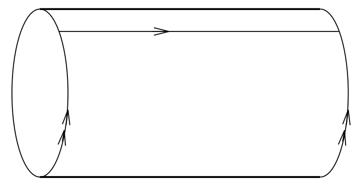
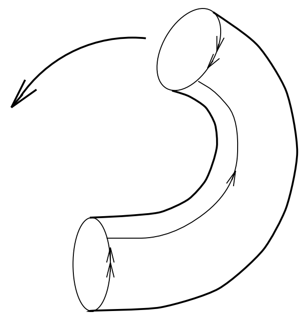
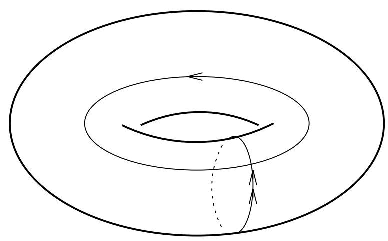
First glue the top edge to the bottom to make a cylinder.



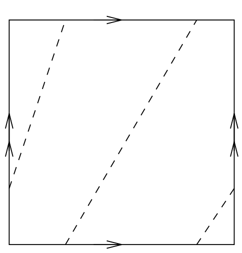
Then bend the cylinder...

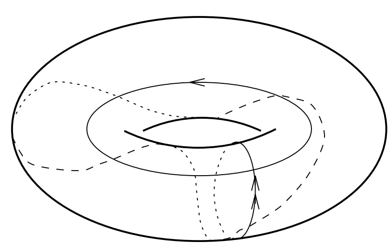


and glue the other edges to complete the torus. The glue lines become the loops shown in Task 2.1.4.



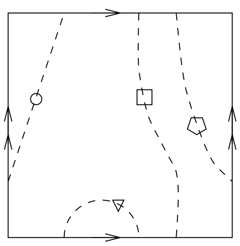
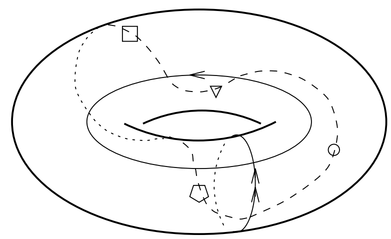
It is easy to draw a square, so we will use a square to represent a torus whenever possible. We call this the **flat torus**. Putting 'arrows' on the sides of a square shows that we mean for the opposite edges to be glued, with the understanding that the glue lines become the two curves shown directly above. Here is an example. Both figures represent the same loop on the torus:





It is worth spending some time looking at that example. On the square, the places where the dotted line hits opposite edges must 'match up.' This ensures that, when the edges are glued, the dotted line will form one continuous loop. To verify that the two pictures represent the same thing, it is easiest to look at where the loop intersects the glue lines. The left picture shows the loop broken into three segments. You need only check that each segment in the right picture is drawn properly.

Here is another example. The segments are marked so that it is easier to see how things correspond.

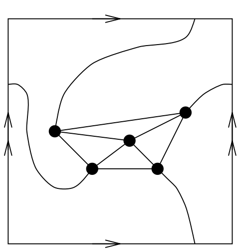
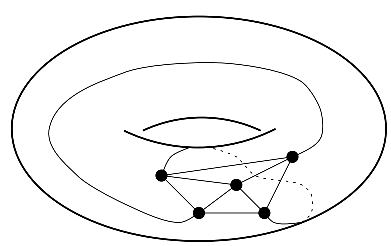
Task 2.2.1: Draw your answers to Tasks 2.1.9 and 2.1.10 on the flat torus.

The next section will provide plenty of practice drawing on the torus.

We mentioned earlier that it is not possible, in general, to distinguish between the two curves shown in Task 2.1.4. Thinking in terms of the flat torus will make this clear. If you repeat the procedure of gluing the square to give a torus, but first glue the left edge to the right edge to make a 'vertical' cylinder, the result will be a torus with glue lines the reverse of those shown above. Since it is impossible to distinguish between the edges of the flat torus, it is impossible to distinguish between the glue lines.

2.3 Graphs on the torus

In the previous chapter we showed that it is impossible to draw the graphs K_5 and $K_{3,3}$ without crossing edges. Actually, that statement isn't quite right. It is impossible to draw those graphs on the *plane* or the *sphere* without crossing edges. We will see that both K_5 and $K_{3,3}$ can be drawn on the torus. Here is one way to draw K_5, given in both representations:

Usually it is easiest to first draw on the square, and then transfer everything to the other picture. When possible, show both pictures.

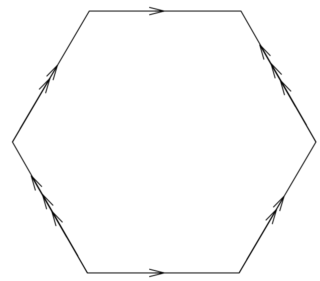
Task 2.3.1: Draw $K_{3,3}$ on the torus. Do the same for $K_{3,4}$ and $K_{4,4}$. There are several nice representations for $K_{4,4}$.

Task 2.3.2: Draw K_6 and K_7 on the torus.

The graphs K_8 and $K_{4,5}$ cannot be drawn on the torus. This is discussed in the next section.

Task 2.3.3: Draw the Petersen graph on the torus.

Task 2.3.4: Gluing opposite sides of a hexagon produces a torus. Use this representation to give a nice way to draw K_7.



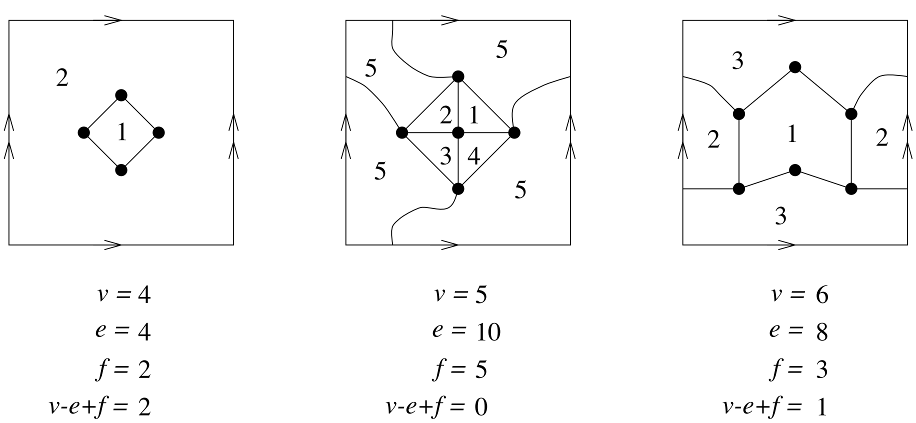
In Section 2.10 we will see why gluing opposite sides of a hexagon gives a torus.

2.4 Euler's formula, again

In the previous chapter we established Euler's formula $v-e+f=2$ for any graph drawn on the plane. This formula also holds for any graph on the sphere.

Task 2.4.1: Explain why $v-e+f=2$ holds for any connected graph drawn on the sphere.

Let's investigate $v-e+f$ for the torus. In these examples it is important to keep track of what is being glued together when counting edges and regions.



This looks like bad news. The value of $v-e+f$ doesn't appear to always be the same. Fortunately, the discrepancy is just an illusion. The key lies in the