
Contents

Preface	xv
Editor Biographies	xxiii
Contributors	xxv
1 Personal Reflections of the SEICCGTC: Origins and Beyond	1
<i>K. B. Reid</i>	
1.1 Introduction	1
1.2 Description of This Chapter	2
1.3 Impressions of the Combinatorial Research Atmosphere in the Late 1960's	3
1.4 Brief Biographies of Early Conference Organizers	6
1.5 Conference Facts	9
1.6 Some Non-Conference Activities at the Conferences	11
1.7 Conference "Firsts"	13
1.8 Some Mathematics from the Fifth Conference (1974)	14
I Combinatorics	19
2 Some of My Favorite Problems (I)	21
<i>Ron Graham</i>	
2.1 Introduction	21
2.2 Prologue	21
2.3 Universal Cycles	22
2.4 Combs	24
2.5 The Middle Binomial Coefficient $\binom{2n}{n}$	26
2.6 The Steiner Ratio Problem	28
2.7 A Curious 'Inversion' in Complexity Theory	30
2.8 A Final Problem	32
3 Variations on the Sequenceable Theme	37
<i>Brian Alspach</i>	
3.1 Introduction	37
3.2 Strongly Sequenceable Groups	40
3.3 Orthogonal Decompositions	41
3.4 Abelian Groups	42

3.5	A Poset Formulation	44
3.6	The Poset Approach	46
3.7	Partial Steiner Triple Systems	47
3.8	Other Decompositions	50
3.9	Sequencing Edges	50
4	A Survey of Stack Sortable Permutations	55
	<i>Miklós Bóna</i>	
4.1	Introduction	55
4.2	Three Equivalent Definitions	56
	4.2.1 The Original Definition	56
	4.2.2 The Original Definition Revisited	56
	4.2.3 The Definition Using Trees	57
4.3	Enumeration Formulas	58
	4.3.1 Exact Formulas	58
	4.3.2 A Surprising Connection with the Pattern 1324	60
	4.3.3 Bounds	61
	4.3.3.1 Stack Words	61
	4.3.3.2 Computing the Upper Bound for $W_3(n)$	63
4.4	The Generating Function of the Numbers $W_l(n)$	65
4.5	Descents	67
4.6	Further Directions	70
5	Dimension for Posets and Chromatic Number for Graphs	73
	<i>William T. Trotter</i>	
5.1	Introduction	73
	5.1.1 Basic Concepts and Results for Dimension	74
5.2	Stability Analysis	76
	5.2.1 Stability Analysis for Dimension	79
	5.2.2 Open Problems for Stability Analysis	81
	5.2.3 Open Problems on Size	82
5.3	Maximum Degree	83
5.4	Blocks in Posets and Graphs	88
	5.4.1 Open Problems involving Cover Graphs	90
6	Erdős Magic	97
	<i>Joel Spencer</i>	
6.1	Introduction	97
6.2	Independent Sets	98
6.3	Avoiding Monochromatic Sets	99
6.4	Six Suffice	102
6.5	QuasiRandomness	104
6.6	Graphons	105
II	Graph Theory	109

7	Developments on Saturated Graphs	111
	<i>Ronald J. Gould</i>	
7.1	Introduction	111
7.2	Saturation Numbers	113
7.2.1	Trees and Forests	114
7.2.2	Cycles	117
7.2.3	Partite Graphs	117
7.3	Limits On The Saturation Function	118
7.4	Hypergraphs	119
7.5	Saturation Spectrum	120
7.6	Variations	124
7.6.1	Weak Saturation	124
7.6.2	Edge-Colored Saturation	127
7.6.3	Other Variations and Results	128
8	Magic Labeling Basics	135
	<i>W. D. Wallis</i>	
8.1	Magic Labeling	136
8.1.1	Labelings	136
8.1.2	The Classical Magic Arrays	136
8.1.3	Magic Labeling	137
8.2	Edge-Magic Total Labelings	138
8.2.1	Basic Ideas	138
8.2.1.1	Definitions	138
8.2.1.2	Some Elementary Counting	138
8.2.1.3	Duality	140
8.2.2	Cliques and Complete Graphs	140
8.2.2.1	Sidon Sequences	140
8.2.2.2	Complete Subgraphs	142
8.2.3	Cycles	143
8.2.3.1	Generalizations of Cycles	143
8.2.4	Complete Bipartite Graphs	143
8.2.5	Trees	144
8.3	Vertex-Magic Total Labelings	145
8.3.1	Basic Ideas	145
8.3.1.1	Definitions	145
8.3.1.2	Basic Counting	145
8.3.2	Regular Graphs	147
8.3.3	Some Standard Graphs	147
8.3.3.1	Cycles and Paths	147
8.3.3.2	Complete Graphs and Complete Bipartite Graphs	147
8.3.3.3	Construction of VMTLs of $K_{m,n}$	149
8.3.3.4	Joins	149
8.3.4	Graphs with Vertices of Degree One	149

9	Block Colorings of Graph Decompositions	155
	<i>E. B. Matson and C. A. Rodger</i>	
9.1	Introduction	155
9.2	Graph Decompositions	159
9.3	Amalgamations and Recent Results	161
9.4	Open Problems	166
10	Reconfiguration of Colourings and Dominating Sets in Graphs	171
	<i>C. M. Mynhardt and S. Nasserar</i>	
10.1	Introduction	171
10.2	Complexity	173
10.3	Reconfiguration of Colourings	174
10.3.1	The k -Colouring Graph	174
10.3.2	Reconfiguration of Homomorphisms	179
10.3.3	The k -Edge-Colouring Graph	180
10.4	Reconfiguration of Dominating Sets	181
10.4.1	The k -Dominating Graph	181
10.4.2	The k -Total-Dominating Graph	184
10.4.3	Jump γ -Graphs	185
10.4.4	Slide γ -Graphs	186
10.4.5	Irredundance	186
11	Edge Intersection Graphs of Paths on a Grid	193
	<i>Martin Charles Golumbic and Gila Morgenstern</i>	
11.1	Introduction	194
11.2	The Bend Number of Known Classes of Graphs	194
11.3	B_1 -Subclass Characterizations	196
11.4	The Strong Helly Number of B_1 -EPG Representations	201
11.5	Algorithmic Aspects of EPG Graphs	202
11.6	Boundary Generated B_1 -EPG Graphs	204
11.7	Concluding Remarks and Further Reading	206
III	Combinatorial Matrix Theory	211
12	A Jaunt in Spectral Graph Theory	213
	<i>Steve Butler</i>	
12.1	Introduction	214
12.2	A Menagerie of Matrices	214
12.2.1	The Adjacency Matrix	214
12.2.2	The Laplacian Matrix and Signless Laplacian Matrix	216
12.2.3	The Probability Transition Matrix and the Normalized Laplacian	218
12.2.4	The Distance Matrix	221
12.2.5	The Seidel Matrix	222
12.2.6	The Quantum Walk Matrix	223

12.3	Strengths and Weaknesses of Different Matrices	223
12.3.1	Combining Spectra	224
12.3.2	Graph Operations	224
12.3.3	A Line Graph Excursion	226
12.3.4	Graphs Determined by Their Spectrum	227
12.3.5	Interlacing	228
12.3.6	Graphs that Have a Common Spectrum	228
12.4	Connectivity	230
12.4.1	Bottlenecks and Cheeger Constants	230
12.4.2	Discrepancy and the Value of Normalizing	231
12.4.3	Ramanujan Graphs	233
12.4.4	Quasirandom Graphs	233
12.5	Starting Your Odyssey in Spectral Graph Theory	234
13	The Inverse Eigenvalue Problem of a Graph	239
	<i>Leslie Hogben, Jephian C.-H. Lin, and Bryan L. Shader</i>	
13.1	Introduction	239
13.2	Ancillary Problems	242
13.2.1	Maximum Nullity and Minimum Rank	243
13.2.2	Variants of Maximum Nullity and Minimum Rank	244
13.2.3	The Minimum Number of Distinct Eigenvalues	245
13.3	Strong Properties and Minor Monotonicity	246
13.3.1	Applications of the Strong Properties	247
13.3.2	Tangent Spaces and the Implicit Function Theorem	250
13.4	Zero forcing, Propagation Time, and Throttling	252
13.4.1	Zero Forcing and Its Variants	252
13.4.2	Propagation Time	255
13.4.3	Throttling	256
13.5	Concluding Remarks and Open Problems	257
14	Rank Functions	263
	<i>LeRoy B. Beasley</i>	
14.1	Introduction	263
14.2	Preliminaries	264
14.3	Matrix Ranks	266
14.4	Rank Functions in Graph Theory	269
14.4.1	Minimum Rank	269
14.4.2	Rank Functions on Graphs Defined by Coverings	270
14.4.3	Rank Functions on Graphs Not Defined by Coverings	272
14.5	Equivalent Rank Functions	272
15	Permutation Matrices and Beyond: An Essay	277
	<i>Richard A. Brualdi</i>	
15.1	Permutation Matrices	277
15.2	Beyond Permutation Matrices	278

15.3	Some Favorite Matrices in these Classes	286
IV	Designs, Geometry, Packing and Covering	291
16	Some New Families of 2-Resolutions	293
	<i>Michael Hurley, Oscar Lopez, and Spyros S. Magliveras</i>	
16.1	Introduction	293
16.2	Preliminaries	294
16.3	Incidence Matrices	295
16.4	The Half-Affine Group	297
16.5	A New Family of 2-Resolutions	297
16.6	Conclusion	299
17	Graphical Designs	301
	<i>Donald L. Kreher</i>	
17.1	Introduction	301
17.2	Graphical Designs	302
17.3	Orbits of S_n Acting on $E(K_n)$	302
17.4	Steiner Graphical Designs	304
17.5	Steiner Bigraphical Designs	310
17.5.1	Remarks on the 5-(16, {6, 8}, 1) Design	311
17.6	Steiner Graphical Designs of Type n^r	311
17.7	Higher Index	312
17.8	Historical Remarks	314
18	There Must be Fifty Ways to Miss a Cover	319
	<i>Charles J. Colbourn and Violet R. Syrotiuk</i>	
18.1	Introduction	319
18.2	Combinatorics of Interaction Testing	320
18.2.1	Covering Arrays	321
18.2.2	Locating and Detecting Arrays	321
18.2.3	Prior Work	322
18.3	A Construction from One-factorizations	323
18.4	Concluding remarks	330
19	Combinatorial Designs and Cryptography, Revisited	335
	<i>Douglas R. Stinson</i>	
19.1	Introduction	336
19.2	The One-time Pad and Shannon's Theory	337
19.3	Threshold Schemes and Ramp Schemes	339
19.3.1	Ramp Schemes	341
19.4	All-or-Nothing Transforms	343
19.4.1	Binary AONT with $t = 2$	345
19.4.2	General AONT with $t = 2$	346
19.5	Algebraic Manipulation Detection Codes	347

19.5.1	Weak and Strong AMD Codes	347
19.5.2	An Application of AMD Codes to Threshold Schemes	348
19.5.3	Combinatorial Analysis of AMD Codes	349
19.5.4	Nonuniform AMD Codes	353
19.6	Conclusion and Open Problems	355
20	A Survey of Scalar Multiplication Algorithms	359
	<i>Koray Karabina</i>	
20.1	Introduction	359
20.1.1	Cryptographic Applications	360
20.1.2	Multidimensional Scalar Multiplication and Endomorphisms	361
20.1.3	Signed Digit Recodings and Differential Additions	362
20.1.4	Side Channel Attacks and Regular Recodings	363
20.1.5	Organization of the Chapter	363
20.2	Variable Scalar and Variable Base	365
20.2.1	Width- w Window Methods	365
20.2.2	Signed Digit Recoding Methods	369
20.2.3	Regular Recoding Methods	372
20.3	Variable Scalar and Fixed Base	375
20.3.1	Split and Comb Methods	376
20.3.2	A Euclidean Type Algorithm	379
20.3.3	Regular Recoding Methods	380
21	Arcs, Caps, Generalisations: Results and Problems	387
	<i>Joseph A. Thas</i>	
21.1	Introduction	387
21.2	k -Arcs of $\mathbf{PG}(2, q)$	388
21.3	Complete Arcs	389
21.4	k -Caps and Ovoids	391
21.5	Ovoids and Inversive Planes	393
21.6	k -Caps and Cap-Codes	394
21.7	k -Caps in $\mathbf{PG}(n, q), n \geq 3$	395
21.8	Generalised k -Arcs and Generalised k -Caps	397
21.9	Generalised Ovals and Ovoids	398
21.10	Regular Pseudo-Ovals and Pseudo-Ovoids	400
21.11	Translation Duals	400
21.12	Characterisations of Pseudo-Ovals and Pseudo-Ovoids	401
21.13	Problems	403
21.13.1	Problems on Arcs	403
21.13.2	Problems on Caps	403
21.13.3	Problems on Generalised k -Arcs and Generalised k -Caps	403
	Index	409