Update to Open Problems in HLA2

April 21, 2014

Section 46.7

Graph Complement Conjecture (GCC)

The following question was not addressed in this section but is natural to ask.

Question Is GCC_F, i.e., $\mathrm{mr}^F(G) + \mathrm{mr}^F(\overline{G}) \leq |G| + 2$, true for fields F other than the real numbers?

The following new example, due to Kathleen Nowak of Iowa State University, shows that $\mathrm{GCC}_{\mathbb{Z}_2}$ is false.

Example 46.7.11 Let G be the graph in Figure 1.

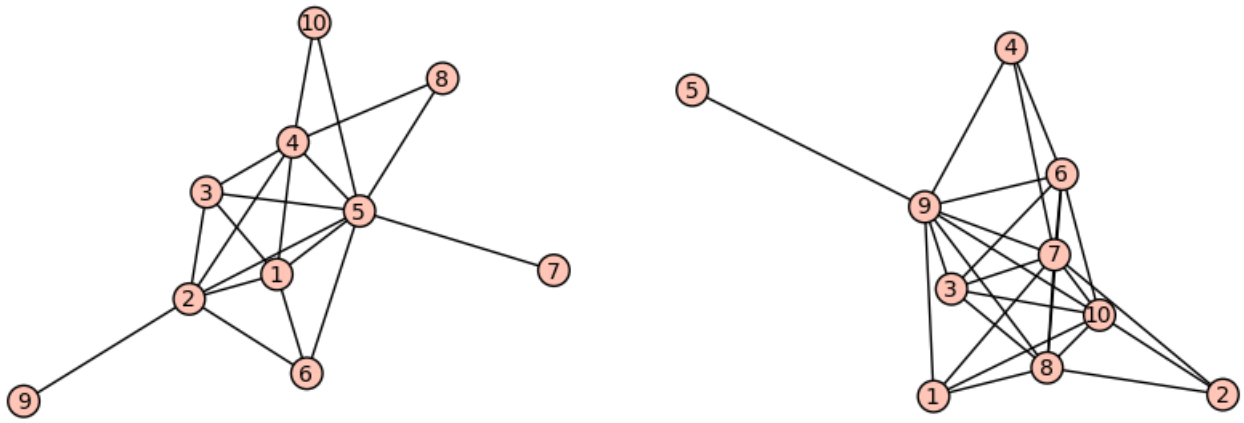

Figure 1: A graph G and its complement $\overline{G}$ that do not satisfy GCC over $\mathbb{Z}_2$.

It is straightforward to verify that $\mathrm{mr}^{\mathbb{Z}_2}(G) = 7 = \mathrm{mr}^{\mathbb{Z}_2}(\overline{G})$; this can be done by computing the ranks of $A_G + D$ and $A_{\overline{G}} + D$ for the 2^{10} possible diagonal matrices D, where A_G denotes the adjacency matrix of G. Thus

$$\mathrm{mr}^{Z_2}(G) + \mathrm{mr}^{Z_2}(\overline{G}) = 14 > 12 = |G| + 2.$$