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Additional useful facts for HLA

Section 2.4

Additional Facts:

1. $A \in F^{n \times n}$ has rank k if and only if A has a nonsingular $k \times k$ submatrix, and every $(k+1) \times (k+1)$ submatrix of A is singular.

2. [GR, 8.9.2] If $A \in F^{n \times n}$ is symmetric and has rank k then A has a nonsingular $k \times k$ principal submatrix.

Section 4.3

Additional Facts:

1. For any matrix $A \in \mathbb{C}^{n \times n}$, the eigenvalues of A are continuous functions of the entries of A. For a simple eigenvalue, the same is true of the eigenvectors (see next fact).

2. [GV96, p. 323] Let $A, F \in \mathbb{C}^{n \times n}$ and $\|F\|_2 = 1$. If λ is a simple eigenvalue of A and $A(\varepsilon) = A + \varepsilon F$, then in a neighborhood of the origin there exist differentiable functions $\lambda(\varepsilon)$ and $\mathbf{x}(\varepsilon)$ such that

$$A(\varepsilon)\mathbf{x}(\varepsilon) = \lambda(\varepsilon)\mathbf{x}(\varepsilon).$$

References

[GV96] G. H. Golub and C. F. Van Loan. *Matrix Computations.* Johns Hopkins University Press, Baltimore, third edition, 1996.