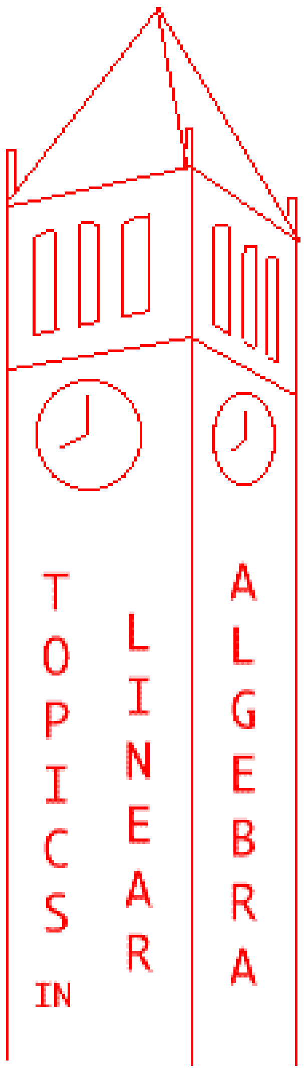


# Topics in Linear Algebra

**A Conference at**

**Iowa State University**

**September 13 – 14, 2002**

# Topics in Linear Algebra

## A Conference at

## Iowa State University

## September 13 – 14, 2002

Sponsored by

Institute for Mathematics and Its Applications (IMA)

International Linear Algebra Society (ILAS)

Iowa State University Department of Mathematics

Iowa State University Department of Statistics

## Organizing Committee


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# Index — Speakers and Abstracts

# Schedule — Topics in Linear Algebra Conference

## Friday, September 13 – Pioneer Room, Memorial Union, ISU

| | |
|---|---|
| 8:00 - 9:00 | Registration — Refreshments |
| 9:00 - 9:10 | Welcome |
| 9:10 - 10:10 | **Sam Hedayat** (ILAS Lecturer) "Extending Saturated D-Optimal Resolution III Two-Level Factorial Designs by Adding More Runs" |
| 10:10 - 10:35 | Huaiqing Wu "Optimal Designs for First-Order Trigonometric Regression on a Partial Cycle" |
| 10:35 - 10:50 | Break |
| 10:50 - 11:50 | **Charles R. Johnson** "Eigenvalues, Eigenvectors, Multiplicities and Graphs" |
| 11:50 - 1:00 | Lunch |
| 1:00 - 1:35 | Shaun Fallat "The Totally Positive Matrix Completion Problem with Few Unspecified Entries" |
| 1:35 - 2:00 | Leslie Hogben "Matrix Completions Methods" |
| 2:00 - 2:25 | Benard Kivunge "The Nonnegative $P_0$-Matrix Completion Problem" |
| 2:25 - 3:25 | Work session on matrix completions |
| 3:25 - 3:40 | Break |
| 3:40 - 4:40 | **Chi-Kwong Li** "Numerical Ranges and Norm Estimation" |
| 4:40 - 5:05 | Edward Poon "Induced Norms, States, and Numerical Ranges" |
| 5:05 - 6:00 | Work session on numerical ranges |
| 6:30 | Conference dinner |

## Saturday, September 14 – Pioneer Room, Memorial Union, ISU

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|---|---|
| 8:00 - 8:30 | Refreshments |
| 8:30 - 9:30 | **David P. Jacobs** "Algorithms, Computation, and Nonassociative Identities" |
| 9:30 - 9:55 | I. R. Hentzel "Automated Idempotent Decomposition Procedures" |
| 9:55 - 10:10 | Break |
| 10:10 - 10:45 | Murray Bremner "Quantization of Lie and Jordan Triple Systems" |
| 10:45 - 11:20 | Alicia Labra "Representations of Train Algebras of Rank Three" |
| 11:20 - 11:45 | Michael Prophet "Shape-preserving Operators and Simplicial Cones" |
| 11:45 - 12:10 | Eric S. Key "Eigenvectors of Products of Companion Matrices" |
| 12:10 - 2:00 | Lunch – Department of Mathematics Picnic |
| 2:00 - 3:00 | Work session on nonassociative algebra |
| 3:00 - 3:25 | Kenneth Driessel "On Computing Eigenvalues Using a Toda Flow with Shifts" |
| 3:25 - 4:25 | **Hans Schneider** "One-Sided Simultaneous Inequalities and Sandwich Theorems for Diagonal Similarity and Diagonal Equivalence of Nonnegative Matrices" |

## *Quantization of Lie and Jordan Triple Systems*

**Murray Bremner**
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This talk will show how the decomposition of the group ring of the symmetric group into a direct sum of full matrix subrings can be used to give a complete classification of n-ary operations. Roughly speaking, row equivalence of matrices corresponds to quasi-equivalence of operations. In particular, the Lie and Jordan products represent the two non-trivial quasi-equivalence classes of binary operations. For ternary operations, there are infinitely many quasi-equivalence classes, which divide into eight classes, and four infinite families of classes each with a single parameter. The Lie triple product is contained in one of the infinite classes, and the other operations in the class can be regarded as quantizations of that product. Similar remarks apply to the Jordan triple product. For special values of the parameter, the operation satisfies an identity of degree 5. This identifies some new ternary operations which define varieties of triple systems, similar to Lie and Jordan triple systems, which seem to be an interesting direction for further research.

## *On Computing Eigenvalues Using a Toda Flow with Shifts*

**Kenneth R. Driessel**
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I shall describe a new flow in the space $Sym(n)$ of real symmetric $n$-by-$n$ matrices which can be used to compute eigenvalues.

For a symmetric matrix $X$, let $X_l$ denote the strictly lower triangular part of $X$. The differential equation $X' = [X_l - X_l^T, X]$ in $Sym(n)$ is called the (generalized) *Toda flow*. This differential equation has the following properties: (1) it preserves eigenvalues; (2) it preserves staircase structure; (3) it converges to diagonal matrices. It follows that the Toda flow can be used to compute eigenvalues of symmetric matrices. It is well-known that the Toda flow is closely related to the QR algorithm without shifts. (See, for example, Demmel [1997] and references therein.)

Recall that if $\lambda$ is an eigenvalue of a square matrix $X$ then $\lambda + s$ is an eigenvalue of the matrix $X + sI$ ; in other words, adding a scalar matrix shifts eigenvalues. I shall describe a differential equation in the space $Sym(n)$ which I call a *Toda flow with shifts* which has the following properties: (1) it preserves shifted eigenvalues; (2) it preserves staircase structure; (3) it converges to diagonal matrices. It follows that this differential equation can be used to compute eigenvalues of symmetric matrices. This differential equation is related to the QR algorithm with shifts.

If there is time I shall also discuss a control theory problem associated with these flows.

*References:*

Demmel, J. [1977], *Applied Numerical Linear Algebra*, SIAM

## *The Totally Positive Matrix Completion Problem with Few Unspecified Entries*

**Shaun M. Fallat**

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A rectangular matrix is called totally positive if all of its minors of all sizes are positive. In this talk we will consider rectangular partial totally positive matrices with exactly one unspecified entry, and characterize the set of positions for that entry that guarantee completability to a totally positive matrix. In each case, the set of completing entries is an open (sometimes infinite) interval. In addition, the pairs of positions that guarantee completability in partial totally positive matrices with two unspecified entries in low dimensions will also be discussed.

## *Extending Saturated D-optimal Resolution III Two-Level Factorial Designs by Adding More Runs*

**A. S. Hedayat**

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We consider the class of saturated main effect plans for the $2^k$ factorial. With these saturated designs, the overall mean and all main effects can be unbiasedly estimated provided that there are no interactions. However, there is no way to estimate the error variance with such designs. Because of this and other reasons, we like to add some additional runs to the set of $(k+1)$ runs in the $D$-optimal design in this class. Our goals here are: (1) to search for $s$ additional runs so that the resulting design based on $(k+s+1)$ runs yields a $D$-optimal design

in the class of augmented designs; (2) to classify all the runs into equivalent classes so that the runs in the same equivalent class give us the same value of the determinant of the information matrix. This allows us to trade runs for runs if this becomes necessary. (3) to obtain upper bounds for determinants of the information matrices of augmented designs. In this paper we shall address these approaches and present some new results.
Remark: This talk is based on a joint work with Haiyuan Zhu.

## *Automated Idempotent Decomposition Procedures*

**Irvin Roy Hentzel**
Department of Mathematics, Iowa State University, Ames, Iowa, USA
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A powerful tool in nonassociative algebra is the Pierce decomposition with respect to an idempotent. The process first finds the eigenvalues of the linear operator representing multiplication by the idempotent. This decomposes the algebra into eigenspaces. Finally, a multiplication table is created between these eigenspaces. We describe a program which first creates the minimal polynomial of this multiplication operator. Then it creates the free nonassociative algebra with idempotent and the appropriate eigenspaces. In this free algebra it computes substitutions in the identities to get relations between two elements from these spaces. This gives the multiplication table between the summands. Then it continues and creates the relations of degree higher than two. The process is interactive. New relations are added to the multiplication process as they are found. The program examines all possible substitutions into the defining identities and prints out the relations in a readable notation.

## *Matrix Completions Methods*

**Leslie Hogben**
Department of Mathematics, Iowa State University, Ames, Iowa, USA
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A pattern is a list of positions in an $n \times n$ real matrix. A matrix completion problem for the class of Π-matrices asks whether every partial Π-matrix whose specified entries are exactly the positions of the pattern can be completed to a Π-matrix. This talk will survey the current state of research on Π-matrix completion problems for many subclasses Π of $P_0$-matrices. Graph-theoretic techniques used to study completion problems will be discussed.

## *Algorithms, Computation, and Nonassociative Identities*

**David P. Jacobs**
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In nonassociative algebra a fundamental problem is the recognition of identities. Indeed, the proofs of most structure theorems are, at some level, based on identities. Given a set I of nonassociative polynomial identities, and a nonassociative polynomial f, we wish to know if f is implied by the members of I. Our talk will discuss computational techniques that have been useful over the last thirty years in attacking the problem. Among these are linear algebra, group rings, sparse matrix representation, dynamic programming, and probabilistic methods. This talk will also recount the experiences and lessons of the Albert project, as well as discuss open questions.

## *Eigenvalues, Eigenvectors, Multiplicities and Graphs*

**Charles R. Johnson**
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We are interested in the very large problem of understanding the possible lists of multiplicities of eigenvalues among Hermitian matrices with a given graph. Here we survey some recent background results and then give some of the very latest results that govern possible lists, especially for trees. The results have some remarkable implications and are also related to the combinatorial structure of eigenspaces. They also give much precise information about eigenvalues with little or no numerical computation. Some examples will be given.

## *Eigenvectors of Products of Companion Matrices*

**Eric S. Key and Hans Volkmer**
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An $n \times n$ matrix $A$ with $n > 1$ is called a *companion matrix* if its entries $A_{i,j}$ satisfy $A_{i,j} = 1$ for $j = i - 1$ and $A_{i,j} = 0$ for $j \neq i - 1$, $i = 2, \dots, n$. It is well known that every eigenvalue of a companion matrix is simple, that is, its geometric multiplicity is equal to 1. The problem we wish to investigate is the following.

Suppose that $A_j,\ j = 1, 2, \ldots, k$, are companion matrices with common eigenvalue $\lambda$. Put $M := A_1 A_2 \cdots A_k$. Under what conditions is $\lambda^k$ a simple eigenvalue of $M$? We shall see that such conditions lead to a recurrence criterion for a class of Markov chains known as Random Walk in a Periodic Environment.

## *The Nonnegative $P_0$-Matrix Completion Problem*


**Benard Kivunge**

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In this paper the nonnegative $P_0$-matrix completion problem is considered. It is shown that a pattern for $4 \times 4$ matrices that includes all diagonal positions has nonnegative $P_0$-completion if and only if its digraph has the property that the induced subdigraph of any 4-cycle is a clique. It is also shown that any positionally symmetric pattern that includes all diagonal positions and whose graph is an $n$-cycle has nonnegative $P_0$-completion if and only if $n \neq 4$.


## *Representations of Train Algebras of Rank Three*


**Alicia Labra and Cristian Reyes**

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Train algebras were introduced by I.M. Etherington[1], as an algebraic framework for treating genetics problems. The aim of this paper is the study of representations and irreducible modules of train algebras of rank 3. We give a characterization of these representations and we prove that every irreducible module has dimension one over the ground field.

Let $F$ be an infinite field of characteristic not 2. A *train algebra of rank 3* is a commutative not necessarily associative algebra $A$ over $F$ satisfying the identity $x^3 - (1+\gamma)w(x)x^2 + \gamma w(x)^2 x = 0$, where $w : A \to F$ is a nonzero algebra homomorphism and 3 is the minimum positive integer for which the identity holds, $\gamma$ is a fixed element in $F$ and $x^t$ is the $t$-th principal power of $x$ defined by $x^1 = x$, $x^{k+1} = x^k x$ for $k \geq 1$.

Let $M$ be a vector space over $F$ and $\mu : A \to End(M)$ a linear map. $\mu$ is said to be *a train representation of $A$ of rank* 3 if the split null extension $A \oplus M$ with homomorphism $\overline{w}$ defined by $\overline{w}(a+m) = w(a)\ \ \forall a \in A, m \in M$ and multiplication given by $(a+m)(b+n) = ab + \mu_a(n) + \mu_b(m),\ \ \forall a, b \in A,\ m, n \in M$ where $\mu_a(m) = \mu(a)(m)\ \ \forall a \in A$ is a train algebra of rank 3. If we put $a \cdot m = \mu_a(m)$, we will say that $M$ is an $A$-module.

We prove the following results:

**Lemma 1.** $\mu$ is a train representation of $A$ of rank 3, if and only if for every $x \in A$,

$$2\mu(x)^2 + \mu(x^2) - 2(1+\gamma)w(x)\mu(x) + \gamma w(x^2)I_M = 0,$$

**Lemma 2.** Let $\mu$ be a train representation of $A$ of rank 3. The

$$\mu(a)\mu(b) + \mu(b)\mu(a) + \mu(ab) = 0 \;\; \forall a, b \in N = Ker(w)$$

and

$$\mu(n)^3 = 0 \;\; \forall n \in N$$

**Theorem 1.** Let $M$ be a finite dimensional irreducible $A$-module. Then the subalgebra of $\mathrm{End}(M)$ generated by $\mu(N)$ is nilpotent.

**Theorem 2.** Every irreducible $A$-module $M$ has dimension one over the ground field $F$.

**REFERENCES**


1. I. M. Etheringhton, *Genetic Algebras* Proc. Roy. Soc. Edinb. 59, 22242-258 (1939);


## *Numerical Ranges and Norm Estimation*


### Chi-Kwong Li

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We describe how to use numerical ranges to give estimates for norms of operators, sums of normal operators, and the derivation operators.


## *Induced Norms, States, and Numerical Ranges*


### Edward Poon

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We show that two induced norms are the same if and only if the corresponding norm numerical ranges and radii are the same, which in turn is equivalent to the vector states and mixed states arising from the norms being the same. This is joint work with C. K. Li and H. Schneider.

## *Shape-preserving Operators and Simplicial Cones*

**Michael Prophet**
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Let $X$ denote a normed linear space, $X^*$ its dual space and $V$ an $n$-dimensional subspace of $X$. Given a closed cone $S^* \subset X^*$ satisfying some additional hypotheses, we say that $f \in X$ *has shape* if $\langle f, \phi \rangle \geq 0$ for all $\phi \in S^*$. Let $S \subset X$ denote the cone of elements with shape and assume $S \cap V \neq \emptyset$. Suppose the linear operator $\widehat{P} : V \to V$ leaves $S$ invariant (i.e., $\widehat{P}(S \cap V) \subset S$). We seek extensions, $P$, of $\widehat{P}$ to $X$ such that $PS \subset S$. We say that such an extension is *shape-preserving*. Consider the statement:

> $\widehat{P}$ admits a shape-preserving extension to $X$ if and only if $S^*_{|V}$ is simplicial.

It has been shown that $\widehat{P} = I_n$ is an operator satisfying this statement. In this talk we will discuss a characterization of operators $\widehat{P}$ satisfying the statement. This characterization involves the eigenstructure of $\widehat{P}$.

## *One-Sided Simultaneous Inequalities and Sandwich Theorems for Diagonal Similarity and Diagonal Equivalence of Nonnegative Matrices*

**Hans Schneider**
Department of Mathematics, University of Wisconsin-Madison, Madison, Wisconsin, USA
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Given nonnegative matrices A,B,C we present results on the existence of diagonal matrices X, Y satisfying $C \leq XAX^{-1} \leq B$ or $C \leq XAY \leq B$. The method is to apply results on simultaneous scalings of the type $XA^{(k)}X^{-1} \leq B^{(k)}$ or $XA^{(k)}X^{-1} \leq B^{(k)}, k = 1, \ldots, s$. Our proofs are graph theoretic. We generalize the setting to matrices with elements in lattice ordered abelian groups with 0. This talk is based on a paper joint with D. Hershkowitz.

## *Optimal Designs for First-Order Trigonometric Regression on a Partial Cycle*

**Huaiqing Wu**

Department of Statistics, Iowa State University, Ames, Iowa, USA
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Trigonometric regression is commonly used to describe cyclic phenomena that occur in the engineering, biological, and medical sciences. Optimal designs for this model on a complete cycle have been studied extensively in the literature. However, much less attention has been paid to the design problem with a partial cycle. This talk addresses this problem for the first-order trigonometric regression. Explicit $D$, $A$, and $E$-optimal designs are analytically derived. These designs are used to evaluate the $D$, $A$, and $E$-efficiencies of the equidistant sampling method commonly used in practice. Efficient and practical designs are then suggested. Some optimal exact designs and optimal designs for all nontrivial subsets of the coefficients are also obtained.

## Participants — Topics in Linear Algebra Conference

| | |
|---|---|
| Brian Birgen | Wartburg College |
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| Bryan Cain | Iowa State University |
| Jane Day | San Jose State University |
| Luz DeAlba | Drake University |
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| Shaun Fallat | University of Regina |
| Corrissa Goertzen | Wartburg College |
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| Sivaram Narayan | Central Michigan University |
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