Implementing algebraic geometry algorithms

The American Institute of Mathematics

The following compilation of participant contributions is only intended as a lead-in to the AIM workshop "Implementing algebraic geometry algorithms." This material is not for public distribution.

Corrections and new material are welcomed and can be sent to workshops@aimath.org

Version: Fri Oct 2 11:13:05 2009

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A.1 Grayson, Daniel

I come as one of the co-developers of Macaulay 2, a software system for computations in algebraic geometry, which we continue to develop.

A.2 Leykin, Anton

I am the organizer of the Numerical Algebraic Geometry part of the workshop and the main developer of the corresponding package in Macaulay2. The questions/issues that will be discussed during this workshop include:

- general structure and documentation of the package;
- solving zero-dimensional polynomial systems;
- tracking user-defined homotopies;
- positive-dimensional solution sets: irreducible and primary decompositions;
- Schubert calculus applications;
- recovering exact data from numerical approximations and certification;
- interfacing with other numerical polynomial homotopy continuation software.

A.3 Petrovic, Sonja

I am generally interested in computational challenges in algebraic statistics, and in developing packages for Macaulay2.

Last year, I participated in the Algebraic Statistics Working Group at SAMSI's year-long program (on Algebraic Methods for Biology and Statistics). As a lead-in to this workshop, I have collected a wish-list of computational tasks from participants of the Working Group. These include minor computational issues that can be addressed quickly, but also some major challenges. A brief list is as follows:

- equivariant Buchberger algorithm
- computing Markov subbases, and studying connectivity of fibers given a subbasis. This should include a nice way to extract a subbasis with given properties (by degree, etc.)
- extending Luis Garcia's Singular library 'Markov.lib' into Macaulay2 and developing the analogue for the Gaussian case
- computing primary decompositions for toric ideals and linking to R?
- a numerical solver for polynomial systems – this links to Jan Verschelde's project
- it seems that there is an interest for interfacing with statistical software R
- identifiability of the parameters in an algebraic statistical model: that is, checking whether the map parametrizing its variety is injective. If not, restrict the parameter space so that it is.
- the last "wish list" comes in form of a list of things available in CoCoA: IdealOfPoints, QuotientBasis and SeparatorsOfPoints as in cocoa; IndicatorFunctions of a finite set of affine points within a larger set. I have not checked any of these; this may be just an issue of translation.

Clearly some of these issues require further discussion. I would be happy to make any progress on any of these topics during the workshop.

A.4 Smith, Gregory

I am particularly interested in iterated normalized Nash blowups for toric varieties and algorithms in toric intersection theory. In preparation for this workshop, I have created a rudimentary Macaulay2 package for working with normal toric varieties;
see http://www.mast.queensu.ca/∼ggsmith/NormalToricVarieties.m2. I hope that we will be able to expand this package (i.e. improve the algorithms, implement new routines, and standardize the function names) during the workshop.

A.5 Taylor, Amelia

I am the organizer of the Algebraic Statistics focus for the workshop and am generally interested in computational algebraic geometry and commutative algebra as related to biology. Sonja Petrovic did a nice job of summarizing a wish list for this area of work along with developing a widely usable package for research involving minimal wiring diagrams and related topics for reverse engineering of biological networks.

A.6 Verschelde, Jan

Following the geometric Littlewood-Richardson rule of Ravi Vakil, homotopy continuation methods are being developed in collaboration with Frank Sottile and Ravi Vakil. An interesting project is to design a good interface to the code in PHCpack and made this interface available as a Macaulay 2 package.