The following compilation of participant contributions is only intended as a lead-in to the ARCC workshop “Arithmetic harmonic analysis on character and quiver varieties.” This material is not for public distribution. Corrections and new material are welcomed and can be sent to workshops@aimath.org

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A.1 Ben-Zvi, David

I am interested in the geometric Langlands program and its interactions with representation theory and topological field theory. I am eager to explore the relation between the harmonic analysis of character varieties and the general Langlands paradigms, in particular possible relations with double affine Hecke algebras.

A.2 Crawley-Boevey, William

I am interested in the representation theory of quivers and algebras, and links with algebraic geometry and noncommutative geometry, typically via moduli spaces of representations. I worked on preprojective algebras of quivers, in which case the moduli spaces are generalizations of quiver varieties, and applied this to solve an additive analogue of the Deligne-Simpson Problem (DSP). The original DSP asked whether or not one can find jointly irreducible matrices in prescribed conjugacy classes whose product is the identity. To tackle this, one replaces quivers by weighted projective lines (in the sense of Geigle and Lenzing), and uses the Riemann-Hilbert correspondence. But many questions remain, and I would like to make some progress with these. There is an analogue of Kac's theorem for weighted projective lines, giving the dimension vectors of indecomposables, but are the numbers of parameters of indecomposables as expected, and is there only one top-dimensional family? What about analogues of Kac's conjectures in this setting, as raised by Schiffmann? What about the DSP in positive characteristic? What about the many other variations of the DSP? In another direction, I would like to better understand how this should all be viewed as noncommutative geometry.

A.3 Harada, Megumi

My recent work is on the topology of symplectic and hyperkahler quotients. More specifically, I study symplectic techniques for explicit computations of the ordinary and equivariant cohomology and integral K-theory of such quotients, of which quiver varieties are examples. In particular, my recent work with Greg Landweber generalizes to the setting of integral K-theory the work of Kirwan (as well many others, e.g. Jeffrey-Kirwan, Tolman-Weitsman, Goldin, etc) for explicitly computing the rational cohomology of symplectic quotients by giving generators and relations. I am interested in whether our techniques in K-theory has applications in the study of quiver varieties.

A.4 Hausel, Tamas

My research interest, relating to the workshop, is the cohomology of the (diffeomorphic) hyperkähler spaces appearing in the non-Abelian Hodge theory of a Riemann surface: $M_{\text{Dol}}$ the moduli space of Higgs bundles, $M_{\text{DR}}$ the moduli space of flat connections on the Riemann surface and $M_B$ the moduli space of characters of representations of the fundamental group of the Riemann surface. Additionally the cohomology of Nakajima star-shaped quiver varieties also appear in these studies, as an open subset in $M_{\text{DR}}$ for a genus 0 Riemann surface with punctures.
In my recent studies some with Villegas and, some with Letellier and Villegas (some related preprints: math.AG/0612668, math.AG/0406380 and math.AG/0511163) the conjectural picture which is emerging is the following: the mixed Hodge polynomial $H(M_B, q, t)$ of $M_B$ possesses some interesting structures.

Conjecture 1: the pure part of $H(M_B, q, t)$ is the Poincare polynomial of the corresponding quiver variety in the indivisible case, and the opposite of Kac’s A-polynomial in the non-indivisible case.

Conjecture 2: the mixed Hodge polynomial $H(M_B, q, t)$ satisfies a Curious Poincare Duality and more generally a Curious Hard Lefschetz Theorem. This in particular implies an isomorphism between the middle cohomology of $M_B$ and the pure part i.e. the cohomology of the corresponding quiver variety by Conjecture 1.

Conjecture 3: The mixed Hodge polynomials $H(M_B, q, t)$ can be described by (explicitly given) generating functions, the right side of which can be considered as certain k-point scalar product of Macdonald polynomials; generalizing the 2-point Cauchy formula which is one of the defining properties of Macdonald polynomials. (I expect that Conjecture 3. can be proved to combinatorially imply the previous two conjectures (except Curious Hard Lefschetz).)

I expect that these three conjectures (and also the spaces $M_{Dol}$, $M_{DR}$ and $M_B$) will have a generalization to other examples of hyperkähler, or more precisely, holomorphic symplectic quotients (such as more general quiver varieties, or holomorphic symplectic quotients obtained from the double of a finite dimensional complex representation of a complex reductive group). In particular with Proudfoot we are studying the analogue of the above picture for toric hyperkähler varieties. Also the case of the tennis-racquet graph on two vertices with an edge between them and a loop on one of the vertices (genus one Riemann surface with one puncture) relate to Hilbert schemes of points on various quasi-projective surfaces, where many of the above conjectures can be explicitly tested.

The tools we are using so far enable us to calculate only the E-polynomial of any of these varieties which is basically the specialization $H(M_B, q, -1)$. We can make these calculations using the character tables of finite groups of Lie type, for now mostly $GL(n, F_q)$. There are also relations to mirror symmetry and Langlands duality (math.AG/0406380) where the character tables of $SL(n, F_q)$ should appear. Those are however less well understood, but the connection to mirror symmetry and Langlands duality makes this an interesting direction to think about.

A.5 Katz, Nick

I have a general interest in arithmetic applications of cohomology.

A.6 Kriloff, Cathy

I am interested in graded Hecke algebras associated, via the definition in [L], to the noncrystallographic reflection groups (i.e., the dihedral groups, type $I_2(m)$, and the symmetry groups of the icosahedron or dodecahedron, type $H_3$, and the dual four-dimensional 600- and 120-cell, type $H_4$). The representation theory of these algebras shares many properties with the corresponding algebras of Lie type but also possesses some interesting differences, as described in [KR,K].

I am currently working with Yu Chen to describe the support of spherical unitary representations for dihedral graded Hecke algebras, and with Arun Ram to establish a
Springer correspondence between tempered irreducible representations of the noncrystallographic graded Hecke algebras and the irreducible representations of the associated reflection group. Recent work of Achar and Aubert establishes a Springer correspondence for the dihedral groups by a different approach, using the Lusztig-Shoji algorithm to compute Green’s functions [AA].

This work has involved generalizing to the noncrystallographic setting algebraic and combinatorial techniques used to study graded Hecke algebras of Lie type. A natural question is whether it is possible to realize representations of noncrystallographic reflection groups in the cohomology of some geometric structure. For this reason I would like to learn more about geometric approaches to representation theory, the varieties that have typically been used, new varieties that are beginning to be considered, and their cohomology. In addition I hope to gain from this workshop a better understanding of Macdonald polynomials and their role in representation theory.

I am also very interested in problems and results connected to the AIM workshop entitled Braid Groups, Clusters, and Free Probability that occurred in 2005 and concerned objects arising in representation theory and other areas which are counted by generalized Catalan numbers. Specifically, the workshop centered around what the relationship might be between ad-nilpotent ideals in semisimple Lie algebras, Garside structures on braid groups, cluster algebras and their generalized associahedra, and free probability. Various combinatorial objects like nonnesting partitions or noncrossing partitions arise in some of these settings and possess interesting generalizations. See [AIM] for resources from the workshop and [CK] for results of a related project.

Bibliography

A.7 Lehrer, Gus

Let X be an algebraic variety (scheme of finite type) over a number field K. Fix a rational prime ℓ. If p is a prime in K, we may consider three cohomology theories for X: de Rham cohomology (over K), analytic cohomology (over C) and ℓ-adic cohomology (over \( \overline{Q}_p \)). All have weight filtrations, but the first 2 also have Hodge filtrations. By canonically identifying these theories, one makes connections between the various filtrations using number theoretic constructions (p-adic Hodge theory). Applications include relations among eigenvalues of Frobenius and Hodge numbers, and the determination of finite group actions on cohomology
by the computation of fixed points of Frobenius. My recent work is an application of these ideas to toric varieties associated with Weyl chambers.

**A.8 Letellier, Emmanuel**

1) With T. Hausel and F. Rodriguez-Villegas we found for each positive integer $n$ a $(q,t)$-table where columns and rows are parametrized by the set of possible “types” of $n$. This table involves $(q,t)$-Kostka polynomials and specializes into the $q$-skeleton of the character table of $GL(n,q)$ and $gl(n,q)$. We constructed this table from the mixed Hodge polynomial of character varieties. I am interested in constructing this $(q,t)$-table directly from exponential sums with “coefficients” in $\mathbb{Z}[q,t]$ which specialize into the exponential sums on $gl(n,q)$ with “coefficients” in $\mathbb{Z}[q]$. These exponential sums with coefficients in $\mathbb{Z}[q]$ are the irreducible invariant characters of $gl(n,q)$ and are related to the irreducible characters of $GL(n,q)$ from my recent work (2005) which was motivated by the work of Kazhdan-Springer (1976), Lusztig (1987), Lehrer (1997) and Waldspurger (2001).

2) Recent notes by Hausel shows an interesting relation between the generating function of the Poincare polynomials of quiver varieties with the Kac-Weyl formula of Kac-Moody algebras. Since the topology of star-shaped quiver varieties is related to the character table of $gl(n,q)$, I am wondering if it could be possible to recover the character table of $gl(n,q)$ from some infinite dimensional algebras. This might give some interesting connections with the classification of finite simple groups since we already know after Borcherds that some sporadic groups (like the Monster) are related to the representation theory of generalized Kac-Moody algebras. We could also investigate the $(q,t)$-deformed case where the Poincare polynomial of star-shaped quiver varieties is replaced by the mixed Hodge polynomial of character varieties.

**A.9 Mason, Sarah**

I am particularly interested in the relationship between Macdonald polynomials and representation theory. My current research involves the combinatorics of a Macdonald polynomial specialization and its related crystal graph structure. I would like to better understand how we can use combinatorial tools to gain insight into the cohomology of character varieties.

**A.10 Mautner, Carl**

I am especially interested in the geometric and representation theoretic aspects of this workshop. Recently, I have been studying perverse sheaves and Hecke algebras and am interested in thinking about using Hecke correspondences to attempt to understand and explain some of the representation theoretic phenomena that appear in the work of Hausel-Rodriguez-Villegas.

**A.11 McGerty, Kevin**

I am currently interested in the modular representation theory of quantum groups (the “root of unity” case) and possible geometric means of studying this theory, via quiver varieties. I am also interested in the theory of character sheaves, and the existence (or lack thereof) of geometrizations of representations and characters.
A.12 Michel, Jean

My interest is in the $\zeta$-function of varieties. More specifically, I have studied the cohomology of Deligne-Lusztig and other varieties using the Lefschetz fixed point theorem. When $X$ is a variety over the algebraic closure $K$ of a finite field endowed with the action of a finite group $G$ defined over a finite subfield $K^F$ (where $F$ is a Frobenius endomorphism), one gets an action of $G \times F$ on the $\ell$-adic cohomology of $X$ which can be studied using the interaction between the character theory of $G$ and the topology of $X$. Of interest are questions of lifting via Shintani descent, and of rationality of the representation on the cohomology (theory of motives).

A.13 Nakajima, Hiraku

My student, Daisuke Yamakawa, recently defines a multiplicative quiver variety, based on earlier works by Crawley-Boevey, Shaw, Hausel and others. Like the case of usual (additive) quiver variety, the multiplicative quiver variety has two parameters, the complex parameter (the value of defining equation) and the real parameter (used to define the stability condition). When the underlying quiver is star-shaped, the multiplicative quiver variety is the moduli space of stable (filtered) local systems over a punctured Riemann sphere. This example is relevant to the Deligne-Simpson problem and has been studied intensively by Crawley-Boevey.

Earlier works mostly study the variety when the complex parameter is generic, but the variety with complex parameter $= 1$ (corresponding to the parameter $= 0$ in the additive case) seems to have more structures, e.g., relation to the representation theory, Lagrangian subvariety, etc, like the additive case. But many things remain to be studied.

A.14 Nevins, Thomas

I am interested in aspects of the geometry and representation theory of double affine Hecke algebras, difference operators, “q-deformed noncommutative geometry,” and Hilbert schemes. I would like to learn more about the work of the other participants in these and related directions.

A.15 Oblomkov, Alexei

I am very much interested in understanding geometry of the character varieties. The most exciting problem for me is to understand for which values of the parameters these varieties are singular. Hopefully we can understand more than that. For example one can hope to describe Gauss-Manin connection for these families of varieties.

A.16 Ostrik, Victor

At this workshop I hope to learn new methods in geometric representation theory. I am particularly interested in computation of cohomology of quiver varieties and character varieties and applications. I am intrigued by possible relations with tensor categories (or topological quantum field theory) and character sheaves.

A.17 Proudfoot, Nicholas

I am interested in cohomology rings of algebraic symplectic quotients, including quiver varieties, character varieties, and hypertoric varieties.
A.18 Ram, Arun

If this workshop can bring me one step closer to understanding the proof of the Weil conjectures, specifically, the proof that push forwards preserve weights (in the form of Deligne’s Weil II) I will be very pleased. I’d like to understand what vanishing cycles, cone, cylinder and the resulting exact sequences “mean” combinatorially.

A.19 Savage, Alistair

My current research focuses on the geometric representation theory of (in particular, finite and affine) Kac-Moody Lie algebras using quiver varieties and Hilbert schemes. I am also interested in the relationship between this geometry and the combinatorics of Macdonald polynomials. During the workshop, I hope to further explore these areas as well as the connections between quiver and character varieties.

A.20 Schiffmann, Olivier

I am interested in the global analogue of Lusztig and Nakajima quiver varieties, in which the category of representations of a quiver is replaced by the category of coherent sheaves on a curve (maybe with some parabolic structure). In this situation, the analogue of Lusztig’s nilpotent variety is given by the Lagrangian fiber in Hitchin’s moduli space of Higgs bundles. When in addition $X$ is of genus zero (resp. one), the role played by the (quantized) enveloping algebra of a Kac-Moody algebra in Lusztig’s theory is now played by a certain quantum loop algebra (resp. a certain quantum elliptic algebra) which may be defined by generators and relations. One is naturally led to the question of constructing an analogue of Nakajima quiver varieties, and finding inside its homology/ K-theory some interesting representations of loop and elliptic algebras.

A.21 Wilkin, Graeme

I am interested in the topology of hyperkahler quotients, in particular SL(n, C) character varieties of surfaces and hyperkahler quiver varieties. Currently I am using Morse theory to study these spaces, and at this workshop I would like to learn more about new techniques for studying these spaces (arithmetic harmonic analysis) and about questions and problems that interest other researchers.

A.22 Webster, Ben

I’m interested in applications of geometry (especially Poisson geometry) to representation theory and knot theory, especially through categorification and canonical bases. Symplectic resolutions of singularities (such as quiver varieties) provide a particularly rich setting for these topics, and characteristic $p$ methods have thus far proved to be very important.