Questions on Model theory for metric structures

1. Let T be a complete superstable metric theory, let σ be a new symbol for a unitary function and let T_σ be the theory $T\cup$ "σ is an automorphism". Assume that T_A, the model companion of T_σ exists. Is T_A supersimple up to perturbations of the automorphism?

   Background: In the first order context, if T is superstable and T_A exists, then T_A is supersimple (Pillay-Chatzidakis). When T is the theory of probability algebras, T is superstable and T_A exists, but T_A is NOT superstable (Ben-Yaacov). But T_A is superstable up to perturbations of the automorphism (Ben-Yaacov, Berenstein).

2. The Ehrenfeucht constructions provide examples of non-countably categorical theories with finitely many countable models.

   a) Is there an analogue of this construction in the metric setting?

   b) We may measure the distance between to models up to perturbation. This gives rise to a pseudometric in the collection of separable models. What can we say about the theory if the quotient of the space of separable structures modulo distance zero is a compact (or finite) space?

   Background:

3. Find connections between different notions of stability: quantifier free stability, stability of structures, stability of theories. What does each of these notions imply?

Background: Assume M is a normed space. If the norm of M is a stable formula, then there is an isometric copy of l_p inside an ultrapower of M for some $p \geq 1$ (Krivine). The group working on "Definable norms and stable Banach spaces"worked on this subject.

4. Let B be a Banach space.

   a) Find the implications of the stability of $Th(B)$.

   b) Find the implications of the $\aleph_1$-categoricity of $Th(B)$.

   Background: The stability of $Th(B)$ implies the existence of isometric copies of l_p in ultrapowers of the models of $Th(B)$ (see previous entry). $\aleph_1$-categoricity of $Th(B)$ implies uncountable categoricity of $Th(B)$ (Ben-Yaacov). Henson conjectured that such uncountable categorical theories are essentially the theory of a Hilbert space.

5. What can we say about theories whose models are isomorphic in the sense of Banach space theory (linearly homeomorphic, not necessarily isometric).

   Background: There is a version of Morley's Theorem for $\aleph_1$-categorical (isometric isomorphism) theories. What can be said when a theory T has a unique model of density chraracter $\aleph_1$ up to linear homeomorphism?

6. Assume a theory is categorical in the sense that any two separable models are linearly homeomorphic. Is there a bound on the coefficients of the deformations?

7. Understand Tsirelson space. Is its theory categorical up to small perturbations?

Background: There was a talk given by William Johnson in this subject.

8. Find examples of $\aleph_1$-categorical structures which are essentially different from Hilbert space.

   Background: See entry 4.

9. Do the randomization of structures due to Keisler generate new examples of categorical or stable structures?

   Background: If the theory of a first order structure is countably categorical, then the theory of its randomization is also separably categorical (Keisler). If the theory is ω-stable then the theory of its randomization is also ω-stable (Keisler). Does it preserve λ-stability? Keisler randomization was studied by the group working on Weak First Order and Probability Structures.

10. Find examples of not essentially separably categorical theories. (Are Nakano spaces an example?)

    Background: The model theory of Nakano spaces is studied in Pedro Poitevin's thesis (UIUC 2006). Nakano spaces generate some theories that are not separably categorical. Are these theories separably categorical up to perturbations of the norm? There was a talk given by C. Ward Henson in this subject.

11. Find examples of stable theories which are not essentially ω-stable theories. (Are Nakano spaces an example?)

    Background: Nakano spaces (see previous entry) generate some theories that are $2^{\aleph_0}$ stable not ω-stable. Are these theories ω-stable up to perturbation of the norm?

12. Find examples of "Non-Modular"stable theories. That is, theories whose beautiful pairs are not essentially separably categorical.

    Background: Probability algebras and Hilbert spaces are ω stable structures whose beatiful pairs are separably categorical. L^p spaces have two non isomorphic beautiful separable pairs, but these two structures are isomorphic up to perturbations of the norm.

13. Build analogues of Hrushovski's constructions in the metric setting.

    Background:

14. Understand the enconding of graphs in metric structures.

    Background:

15. Study the group configuration theorem in the setting of continuous logic.

    Background:

16. Study the existence of ample generics in the setting of ω-stable separably categorical continuous theory.

    Background: Generalize the ideas of Hodkinson, Hodges, Lascar and Shelah presented in the paper "The small index property for ω-stable, ω-categorical structures and the random graph"to the setting of metric structures.

17. Is there a model theoretic understanding of parametric families of probability measures. In particular, find ways to compare different experiments.

    Background:

18. Find connections between different notions of genericity. In particular, the notion of genericity of Vershik and the notion of being existentially closed. Urysohn space is generic in both senses.

    Background:

19. Study the definable sets in the expansion of the Urysohn space with a predicate for an involution map.

    Background:

20. Study non commutative probability spaces.

    Background: The working group on Non Commutative Probability Spaces worked in this subject.

21. Find examples of continuous theories with the NIP.

    Background: If M is first order structure whose theory has the NIP, does the randomization of M have the NIP?

22. Understand basic results of stable/ω-stable groups in the setting of continuous logic.

    Background: The working group on Stable Groups worked in this subject.

23. Study definable sets.

    Background: The working group on Definable Sets worked in this subject.

24. Let $\mathcal{L}$ be a metric language. A weak $\mathcal{L}$-structure is a 2-sorted structure $(M, [0,1], ...)$ where we have the usual interpretation for all the symbols in $\mathcal{L}$ and we add a symbol for every continuous function from $[0,1]^n$ into $[0,1]$. We work with non-standard models of the theory of such a

structure and consider the metric structure obtained by taking the standard part map. Is there a justification of why one uses the quantifiers sup, ínf? Are there clear proofs of the keisler-Shelah Theorem or the omitting types Theorem?

Background: The working group on Weak First Order and Probability Structures worked in this subject.

25. Understand integrals as oppose to types in continuous logic.

    Background:

26. Under which circumstances are there enough definable sets? (i.e. under which assumptions are types determined by the definable sets they belong to?). In particular, do we get a positive answer when T is ω-stable?

    Background:

27. Find a mechanism for deducing an "almost isometry theorem"from an isometric theorem using compactness (an ultraproduct).

    Background:

28. Is there a notion of "explicitly defined norm" so that any Banach space with an explictly defined norm hereditarily contains an l_p space?

    Background: The group working on Definable Norms and Stable Banach Spaces worked on this subject.

29. Study asymtotic cones.

    Background: The group working on Asymptotic cones worked on this subject.