AIM Workshop June 25, 2007. Afternoon discussion.

1. How to compute KL (Kahzdan-Lusztig) polynomials, what has been done, and what is available?
   (a) GAP, Chevie (can deal with the affine Weyl groups), in small cases.
   (b) Programs of Fokko du Cloux.

Parabolic KL polynomials are the "essential" KL polynomials for modular cases. There are two types of parabolic KL polynomials: the M and N polynomials introduced in Lin's morning talk.

2. What is the relation between radical filtrations (e.g., as in Lin's talk) and KL polynomials? Does the radical filtration determine the KL polynomials? What if you have knowledge of the extensions between standard modules and irreducible modules (or between irreducible modules and co-standard modules)?

3. How do KL polynomials determine the ℓ-decomposition numbers for $GL_n(q)$?

4. KL polynomials for singular weights instead of regular weights.

The original Lusztig conjecture was simply a character formula. The version involving dimensions of Ext groups stated by Lin in the morning talk is a later formulation. The original statement of the conjecture involved KL polynomials evaluated at 1.

5. Lin's conjecture

$$[\mathrm{Rad}^i \,/\, \mathrm{Rad}^{i+1}\, T_\lambda^\mu M_{y\cdot\lambda} : T_\lambda^\mu L_{w\cdot\lambda}] = [\mathrm{Rad}^i \,/\, \mathrm{Rad}^{i+1}\, M_{y\cdot\lambda} : L_{w\cdot\lambda}]$$

   The conjecture has been verified for rank 2 groups.

6. What happens with standardly-stratified algebras?

7. Can we compute the radical filtrations of the Weyl modules, and what tools are there for making the computations? Same question for the Jantzen filtration. Calculations of the Jantzen filtration have been done for "small" dimensional cases.

8. Radical length of the Weyl module.

9. In almost simple groups of Lie type, what are the smallest and second-smallest conjugacy classes?

10. In cross-characteristic representations, what is the smallest characteristic in which it can be realized?

11. When does one simple group sit inside another?

12. When does an irreducible representation of a simple group restrict to an irreducible representation of a simple subgroup?

13. Given an irreducible representation of a finite group, how do you tell when it is in a classical group?

14. Role of KL polynomials in characteristic ℓ vs. characteristic p.

15. Overview of where computing is right now. Why would you use one system over another, or why it wouldn't matter. What algorithms are there/aren't there? In what areas should we invest computational efforts? What are our computational needs for the future (in a general sense)?

16. The real Lie groups project.

17. The "categorification" process. (Example in Jantzen's book on nilpotent orbits, Chapters 1-2.)

18. Computational work done on basic algebras for algebras associated with module categories of bounded highest weight for algebraic groups in characteristic p.

19. Connections between KL polynomials and quivers / relations.

20. When does a Specht module have simple socle?

21. Compute the number of irreducible components (equivalently, the number of maximal conjugacy classes) of the spectrum of $H^*(GL_n(p)_k)$.

22. Compute examples of support varieties of Weyl modules for higher Frobenius kernels G_r, $r > 1$.

23. Explicit descriptions of group algebras for G_r.

24. How do group algebras sit inside matrix algebras? (integral or p-adic cases)

25. Calculations of quantum groups at p^n-roots of unity, $n > 1$.

26. Given extensions between simple modules D_λ, D_μ for S_n, do λ, μ have to be comparable?

27. How might you try to compute the cohomology of finite groups? What are the current size limitations?

28. What are the "nilpotent" elements in small quantum groups? How do you go about finding them? Same question for $u_q(sl_3)$.

29. The `quagroup` package for GAP.

30. Ext groups between Weyl modules and irreducible modules for very big weights for SL_2.

31. 5-10 minute lecture demo on wikis.

32. Lusztig conjectures for Lie algebras in positive characteristic.

33. Analogous highest-weight categories, double affine Hecke algebras, complex reflection groups, Cherednik algebras. Some of the above questions in the more general context.

34. Methods for presenting computational data (especially on the internet). Modifying existing programs to make data more presentable. (Is there funding for this?)

35. Demonstrations of packages for GAP, Chevie, Lie.