High and low forcing

The American Institute of Mathematics

The following compilation of participant contributions is only intended as a lead-in to the AIM workshop "High and low forcing." This material is not for public distribution.
Corrections and new material are welcomed and can be sent to workshops@aimath.org
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A.1 Aspero, David

I would like to report on recent work on symmetric systems of models of two types and their applications so far.

I am also seeking to gain a deep understanding of the other current trends in the area of forcing with side conditions, broadly understood, and also to familiarize myself with some of the current techniques for forcing using large cardinals.

A.2 Cox, Sean

I am familiar with the "low" side of the workshop topic, and would like to learn more about the "high" side. For example, I have worked quite a bit with side condition forcings, in particular to investigate principles related to so-called "guessing models" and supercompact tree properties (much of this was joint with John Krueger). There are several questions about guessing models which likely involve singular cardinal combinatorics, and which might be good topics for this workshop.

A.3 Fontanella, Laura

I am interested in consistency results obtained from the method of forcing, large cardinals axioms, and infinitary combinatorics.

A.4 Friedman, Shoshana

My research is in forcing and large cardinals and I am currently interested in exploring the ways to force V not equal HOD in models with large cardinals. For example, via failures of "covering" or HOD-supercompactness. Also, to what extent failures of δ covering (and approximation) give large cardinal strength. If any.

I am not particularly familiar with the forcing techniques that are the focus of the workshop. However, I suspect they might yield some promise in the areas I mentioned above, so my aim is to become more familiar with them to determine more specifically how.

A.5 Gitman, Victoria

My expertise is in the areas of forcing and large cardinals, particularly, in indestructibility properties of large cardinals, where the two subjects interact. I am not an expert in the two specific areas on which the workshop is focused, but I am familiar with the techniques they employ. Therefore my main goals for the workshop are to first get a deeper understanding of these two topics and then to see how my particular knowledge can contribute to making progress on the open questions.

A.6 Hayut, Yair

A long standing open question, which I am interested in, is the consistency strength of the Suslin hypothesis at $\aleph_2$. Shelah and Laver showed that it is consistent, relative to a weakly compact cardinal, that $2^{\aleph_1} = \aleph_3$ and there are no Suslin trees at $\aleph_2$, thus a weakly compact is an upper bound for $\mathbf{SH}_{\aleph_2}$.

The methods of forcing with size conditions might be helpful in the attempts to answer the following questions:

- Can one obtain a model of $\mathbf{SH}_{\aleph_2}$, without using large cardinals?
- Is $\mathbf{GCH} + \mathbf{SH}_{\aleph_2}$ consistent relative to large cardinals?

A.7 Krueger, John

I am primarily interested in finding new ideas on applying side condition methods to obtain consistency results. I am especially interested in applying two of the frameworks which I have developed, namely, coherent adequate forcing and S-obedient side conditions. The former framework was shown by M.A. Mota and myself to preserve CH, and so could be useful for obtaining consistency results involving CH.

My secondary topic of interest would be combinatorics on the successor of a singular cardinal. This includes studying the approachability ideal and also the existence or nonexistence of partial squares.

A.8 Mildenberger, Heike

I am interested in preservation theorems for relations on the reals that are defined by names coming from former stages of the iteration.

A.9 Moore, Justin

I would like to better understand Itay Neeman's solution to Baumgartner's problem. In particular, I would like to better understand his technique of using countable families of models as side conditions to iterate σ-closed posets.

The following are open problems which seem like good test questions for this methods strengths and limitations:

Is it consistent with CH that every ω_2-Countryman line is minimal?

Is it consistent with CH that every two ω_2-Countryman lines are near or co-near?

Here an ω_2-Countryman line is a linear order of cardinality greater than ω_1 whose square is the union of ω_1 chains (in the coordinate-wise order).

A more ambitious question is:

Is it consistent that every ω_2-Aronszajn line contains an ω_2-Countryman line?

The first two questions are higher cardinal analogues of consequences of MA_{ω_1}. Note however that the conjunction of CH and the forcing axiom for ω_1-centered (in particular ω_2-c.c.) σ-closed posets is inconsistent (if we are required to meet ω_2-dense sets).

A.10 Raghavan, Dilip

I have recently been working on several questions in partition calculus. Forcing techniques at both large and small cardinals are required to treat such questions. I hope to better understand recent advances in forcing methods which singularize large cardinals as well as advances in iterating σ-closed partial orders. Here is a typical example of a question I am interested in:

Is it consistent relative to large cardinals that CH holds and $\omega_2 \to (\omega_2, \alpha)^2$, for all $\alpha < \omega_2$?

A.11 Viale, Matteo

My current interests are focused on the relation between forcing axioms and generic absoluteness results. Forcing axioms are often used to solve problems as they provide strong means to get existential witnesses of properties which otherwise one wouldn't be able to realize. I've been searching for logical grounds on which to base the effectiveness and success forcing axioms have met. I believe these logical grounds can be given by generic absoluteness results.

Roughly a generic absoluteness results states that for formulae ϕ of a certain logical complexity L (for example projective formula, Borel predicates, etc....), a theory T extending ZFC, and for a class Γ of forcing notions definable in T, the following are equivalent over any theory S extending T:

- $S \vdash \phi$,
- S proves that some forcing in Γ realizes ϕ and T at the same time,
- S proves that all forcings in Γ which realize T also realize ϕ.

One can view such type of results either as proof boosters (the first two equivalences show that in order to prove ϕ form S it is enough to prove from S that ϕ and T are jointly compatible by a forcing in Γ), or as a completeness theorem (the first and the third equivalences assert that boolean valued models in Γ provide a complete semantic for L with respect to any S extending T).

The basic absoluteness results is the preservation of Δ_1-properties in all forcing extensions, the first non-trivial (and probably most useful) generic absoluteness result is Shoenfield's absoluteness for Σ^1_2-properties, the pioneering and major results in this area are Woodin's proof of the generic absoluteness of the the theory of $L(ON^\omega, UB)$ assuming the existence of class many Woodin limit of Woodins (UB is the class of universally Baire sets of reals).

My recent work outlines that Woodin's results can be extended so to explain also the success of forcing axioms such as MM and PFA (among many other forcing axioms). In a series of works by myself (Category forcing, MM^{+++} and generic absoluteness for strong forcing axioms), with Giorgio Audrito (Absoluteness via resurrection), and with David Aspero (in preparation) I can now show that to any well behaving class of forcing Γ one can attach a cardinal κ_Γ and natural axioms $CFA(\Gamma)$ (or $RA_\omega(\Gamma)$ which have the following properties:

- $CFA(\Gamma)$ implies $RA_\omega(\Gamma)$.
- $CFA(SSP)$ implies MM, $CFA(proper)$ implies PFA, $RA_\omega(SSP)$ implies BMM, $RA_\omega(proper)$ implies $BPFA$, etc....
- $CFA(\Gamma)$ yields the generic absoluteness result for the theory of $L(ON^{\kappa_\Gamma})$ with parameters in $P(\kappa_\Gamma)$ with respect to forcing in Γ which preserve the axiom $CFA(\Gamma)$.
- $_\omega(\Gamma)$ yields the generic absoluteness result for the theory of $H(2^{\kappa_\Gamma})$ with parameters in $P(\kappa_\Gamma)$ with respect to forcing in Γ preserving $RA_\omega(\Gamma)$.
- $RA_\omega(\Gamma)$ is consistent relative to the existence of a Mahlo cardinal for most classes Γ.
- $CFA(\Gamma)$ is consistent relative to the existence of a super huge cardinal for most classes Γ.

More specifically:

- To prove the consistency of $RA_\omega(\Gamma)$ one needs an iteration theorem stating that any iteration of posets in Γ (with all quotients in Γ) has a limit in Γ and that this limit can be the direct limit if the iteration has length κ_Γ.
- To prove the consistency of $CFA(\Gamma)$ one needs moreover that the category Γ has the freezeability property. To state it observe that the category Γ with complete

homomorphisms with a quotient in Γ can also be seen as a class poset ordered by these complete homomorphisms. Now the freezeability property plus the iteration property above yield that there is a subcategory D of Γ which is a dense suborder of the partial order Γ and such that with the arrows inherited from Γ it is a directed graph (at most one arrow between two objects in D).

It can be of interest to see whether the iteration theorems introduced by Neeman and preserving ω_1 and ω_2 apply to a class of forcing Γ satisfying any of the above requirements to yield the corresponding $CFA(\Gamma)$ or $RA_\omega(\Gamma)$.

A.12 Zeman, Martin

My interests in the workshop lie in (a) forcing methods for adding closed unbounded subsets of cardinals and iterating forcing posets of this kind, (b) singularizing caradinals without disturging the universe too much, in particular using Prikry type focings and tree forcings, and (c) aplications of the above methods methods to infinitary combinatorics, in particular in study of ideals on small cardinals, predominantly the non-stationary ideal, square principles, and diamond principles.