Open Problems at the Random Analytic functions Workshop at AIM

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Abstract

This is a compilation of the open problems posed by the participants of the AIM Workshop on Random Analytic functions.

Question 1: Wenbo Li Consider random polynomials in one variable (real or complex). Find the asymptotics of the norm of the largest zero as the degree of the random polynomials tends to infinity. \diamond

Question 2: Yan Fyodorov What is the mean density of permanental polynomials ? This is unknown for random matrices of size greater than 5×5 .

Question 3: Ashkan Nikeghbali What can one tell about the distribution of the zeros of the derivative of characteristic polynomial of random unitary matrix, especially near the boundary of the unit circle. Also what can one tell about $\mathbb{E}[|f'|^s]$ as s tends to zero ? \diamond

Question 4: Maurice Rojas Investigate the connections between random sparse polygons and Newton polytopes. This should be extended to the case of random Viro diagrams. \diamond

Question 5: Balint Virag Find a Gaussian entire function with negatively correlated zeros. We know that there exists such a function on the unit disc. This is related to the repulsion properties of random polynomials. \diamond

Question 6: Bernard Shiffman Does the Fubini-Study metric on \mathbb{P}^1 minimize the expected number of critical points? There are reasons to conjecture that the answer is Yes. \diamond

Question 7: Steven Evans Is there a necessary and sufficient condition for a given correlation function to be the correlation function of an entire Gaussian function? \diamond

Question 8: Maurice Rojas What is the probability that a random Viro diagram contains no sphere? \diamond

Question 9: Ashkan Nikeghbali What are the natural physical examples of random functions with GUE zeros on the real line? \diamond

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Question 10: Scott Sheffield Consider a function who Fourier transform is white noise on the unit circle. We aim to understand the web like appearance, that is the zero level lines of the Gaussian free field. Zelditch and Schramm make the above question more precise. \diamond

Question 11: Yan Fyodorov Given a random entire function on order 1, real on real line with given distribution of real zeros, what is the distribution of zeros of $f ? \diamond$