Hecke correspondance

(G)

- Rk 2 v. bundle over X Rieman surface

  E stable $\forall L \subset E$ $\deg L < \frac{1}{2} \deg E$

(1) M_0 = fixed trivial determinant $= Rep(\pi_1, SU(2))$
singularités

(2) M_1 = fixed determenent = $G(p_1)$

$\left\{ \begin{array}{l} \text{trivial bundle} \\ \text{change of coordinate} = z \end{array} \right\}$.

- $M_1 = \{ \rho : \pi_1(X \setminus p) \to SU(2) \}$. smooth

  $Hol(\rho) = -1$

$\downarrow$

$Hom(\pi_1(X), SO(3)) / SO(3)$

$\nwarrow$ Narasimhan - Seshadri

(3) Let $M_\alpha = \{$parabolic stable rank 2 - bundle$\}$

$\alpha \in [0,1]$

* parabolic structure at p:

$\ell_p \subset E_p^*$, choice of the line

* stability: (E, ℓ_p, α) is stable.

if for all lines bundle $L \subset P$.

$\deg L \pm \alpha < \frac{1}{2} \deg E$ $\quad + (1-\alpha)$

depending if $\ell_p | L_p = 0$ or not

$M_\alpha = \{ \rho : \pi_1(X \setminus p)$ $\rho\left(\prod_{j=1}^{g} [A_i, B_j]\right)$

Mehta-Seshadri Biquard $\nearrow$ $= (e^{-2\pi i \alpha}, e^{2\pi i}$,

(4)
$$\begin{tikzcd} & M_\alpha \arrow[dl, "\pi_0"'] \arrow[dr, "\pi_1"] & \\ M_0 & & M_1 \end{tikzcd}$$
correspondence.

we have two projections as above obtained using the complex structure.

construction of π_1

we fix E so that $\det(E) = \mathcal{O}(p)$ (6)

$$(E, \ell_p, \alpha) \to E$$

$$M_\alpha \longrightarrow M_1$$

this projection is forgetful.

construction of π_0^-

• Let $0 \to F \to E \xrightarrow{\ell_p} \mathbb{C}_p \to 0$

F is bundle

around p you consider $\begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix}$

then F is semisimple.

gradient flow of the yang mills actually realizes these projection – (Daskalopoulos - Wentworth).

Hitchin fibration

$$\mathcal{M}_H(G) = \{E, \varphi\}$$

φ = higgs fields

$$\left\{ \begin{array}{l} \mathcal{M}_H(G) \\ \varphi \end{array} \right. \longrightarrow \det(1 - t\varphi) \in \bigoplus_i H^0(K^i)$$

Hitchin fibration.

Let ω be the Hitchin symplectic form, on $\mathcal{M}_H(G)$.

Hitchin: this is a completely integrable system with respect ω

fibers are abelian varieties.

the Hitchin symplectic form

• identifies "$\mathcal{M}_H(G)$ with $T^*\mathcal{M}(G)$"

and the hitchin symplectic form by the classical construction

$$\mathcal{M}_H(G) \supset T^*M_G$$

open "dense"

Langlands correspondance and Hitchin fibration

$$\begin{array}{ccc} \mathcal{M}_H(G) & & \mathcal{M}_H(\check{G}) \\ & \searrow \quad \swarrow & \\ & H^0_\bullet(K) \oplus \cdots \oplus H^0_\bullet(K^{n}). & \end{array}$$

then the « fibers are dual abelian varieties » - proved by Hausel - Thaddeus .... - Donnelly - Pantev

..-.

<u>Spectral curves</u> $G = GL_n$.

(E, φ); $\varphi \in H^0(K \otimes \mathrm{End}(E))$,

$$\det(x - \varphi) = x^n + p_1(\varphi)x^{n-1} + \cdots + p_n(\varphi).$$

$$\varphi \xrightarrow{H} (p_1(\varphi), \cdots, p_n(\varphi))$$

is the Hitchin fibration.

(G)

So now let.

$$\begin{array}{ccc} x & \xrightarrow{\quad\sigma\quad} & x^n + p_1(\varphi)x^{n-1} \cdots + p_n(\varphi) \\ \cap & & \cap \\ K & & K^n \end{array}$$

$\sigma: \in \Gamma$

So the spectral curve Σ_1,

$$\Sigma_1 = \{x \in K,\ \det(x - \varphi) = 0\}$$
$$\downarrow \pi$$
$$S$$

this is a ramified covering.

there is a line bundle $U \to \Sigma_1$ such that $\pi_* U = E$.

<u>spectral curve</u>: $\Sigma_1 \hookrightarrow K$, with a line bundle U (i.e a point in $\mathrm{Jac}(\Sigma_1)$.) Therefore the fiber is an algebraic variety.

6

$$H^{-1}(p_1, \dots, p_n) = \mathrm{Jac}(\tilde{\Sigma}_1)$$

Case of $SL(n,\mathbb{R})$ $\quad p_1 = 0$

we end up with an additional condition,

$$\det(\pi_* U) = \text{trivial bundle}.$$

this is understood in terms of algebraic geometry.

Let $N_m \left(\sum_1 n_i x_i\right) = \sum_1 n_i \pi(x_i)$.

$$N_m : \mathrm{Jac}(\tilde{\Sigma}_1) \to \mathrm{Jac}(S).$$

Condition

$U \in N_m^{-1}\{0\}$ = Prym variety is an algebraic variety.

Case of $PSL(n,\mathbb{R})$

we have an identification on $SL(n,\mathbb{R})$ bundle; $PSL(n,\mathbb{R})$ bundles are equivalence class

$\rightsquigarrow U \sim V$ if $U = V \otimes \pi^* L$

line bundle over the spectral curve.

hence the fiber of the Hitchin fibration is identified with

$$\mathrm{Jac}(\tilde{\Sigma}_1) / \pi^* \mathrm{Jac}(S)$$

dual to the Prym variety

$$N_m^{-1}(0).$$

langlands duals:

maximal torus for G

$\wr$

dual maximal torus ${}^L G$