

Hecke  
correspondence

• RR 2 v- Bundle over X Riemann surface.

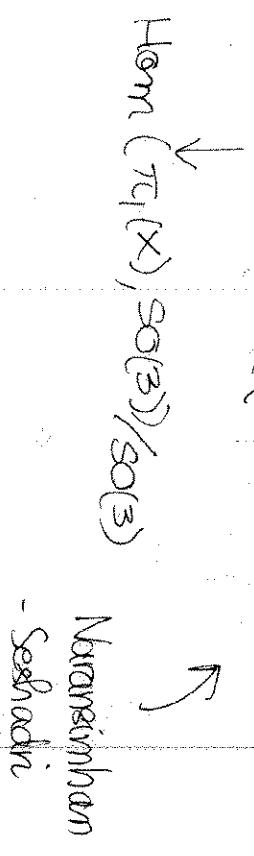
Estable  $V \subset E$   $\deg L < \frac{1}{2} \deg E$

①  $M_0 =$  fixed trivial =  $\text{Rep}(\pi_1, \text{SU}(2))$  determinant singularities

②  $M_1 =$  fixed determinant =  $G(p_1)$  of trivial Bundle change of coordinates =  $\mathbb{Z}$

•  $M_1 = \{ p : \pi_1(X \setminus p) \rightarrow \text{SU}(2) \}$  smooth

$\text{Hom}(p) = -1$



③

③ let  $M_\alpha = \{ \text{parabolic stable rank 2-bundle} \}$   
 $\alpha \in \mathbb{R}/\mathbb{Z}$

\* parabolic structure at  $P$ :  
 $E \subset E_P^*$ , choice of the line

\* stability:  $(E, P, \alpha)$  is stable.

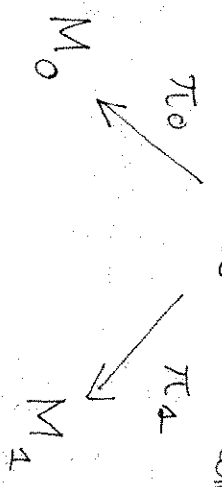
if for all lines Bundle LCP  
 $\deg L \neq \alpha < \frac{1}{2} \deg E$

depending if  $E_P \perp L_P = 0$   
 or not

$M_\alpha = \{ p : \pi_1(X \setminus p) \rightarrow \rho(\pi_1[A_1, B_1])$

Melita-Seibachin Biquard  $\rightarrow = (e_{-2\pi i \alpha}, e_{2\pi i \alpha})$

④  $M_\alpha$  correspondence.



We have two projections as above obtained using the complex structure.

construction of  $\mathcal{Y}_1$   
 we fix  $E$  so that  $\det(E) = \mathcal{O}(p)$  ③

$$(E, \mathcal{O}_p, \alpha) \rightarrow E$$

$$M_\alpha \rightarrow M_1$$

This projection is forgetful.

construction of  $\tau_0$

$$\bullet \text{ set } 0 \rightarrow F \rightarrow E \xrightarrow{\rho} \mathcal{O}_p \rightarrow 0$$

$\mathcal{N}$  is bundle

$$\text{around } p \text{ you consider } \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix}$$

then  $F$  is semi-simple

$\{$  gradient flow - actually realized these

$\{$  projection - (Daskalopoulos - Wentworth)

Hitchin fibration

$$\mathcal{M}_H(G) = \{E, \varphi\}$$

"  
 Friggs fields

$$\int \mathcal{M}_H(G) \xrightarrow{\varphi} \det(1 - t\varphi) \in \bigoplus_i H^0(K^i)$$

Hitchin fibration

Let us be the Hitchin symplectic form on  $\mathcal{M}_H(G)$ .

Hitchin: this is a completely integrable system with respect to  $\omega$   
 $\{$  fibers are abelian varieties.

the Hitchin symplectic form

- identifies  $\mathcal{M}_H(G)$  with  $T^* \mathcal{M}_H(G)$  and the Hitchin symplectic form by the classical construction

$$\mathcal{M}_H(G) \supset T^* M_G$$

open  $\hookrightarrow$  dense

Langlands correspondence and Hitchin fibration.

$$\mathcal{M}_H(\mathcal{G}) \quad \mathcal{M}_H(\check{\mathcal{G}})$$

$$\swarrow \quad \searrow$$

$$H^0(K) \oplus \dots \oplus H^0(K^{2g})$$

From the fibers are dual abelian varieties  $\rightarrow$  - proved by Hausel-Thaddeus - Donnelly-Pantev.

Spectral curves  $G = GL_m$

$$(E, \varphi), \quad \varphi \in H^0(K \otimes \text{End}(E))$$

$$\det(x - \varphi) = x^m + p_1(\varphi)x^{m-1} + \dots + p_m(\varphi)$$

$$\varphi \mapsto (p_1(\varphi), \dots, p_m(\varphi))$$

is the Hitchin fibration.

③

So now let.

$$\begin{array}{ccc} x & \longrightarrow & x^m + p_1(\varphi)x^{m-1} + \dots + p_m(\varphi) \\ \uparrow \cap & & \uparrow \cap \\ K & \xrightarrow{\sigma} & K^m \end{array}$$

$$\sigma \in \Gamma$$

So the spectral curve  $\Sigma^1$ ,

$$\Sigma^1 = \{x \in K, \det(x - \varphi) = 0\}$$

$$\downarrow \pi$$

This is a ramified covering.

There is a line bundle  $\mathcal{O} \rightarrow \Sigma^1$

and that  $\pi_* \mathcal{O} \cong E$ .

Spectral curve:  $\Sigma^1 \hookrightarrow K$ , with

a line bundle  $\mathcal{O}$  (i.e. a point

in  $\text{Jac}(\Sigma^1)$ ). Therefore the fiber

is an algebraic variety.

③

$$H^{-1}(P_1, \dots, P_m) = \text{Jac}(\Sigma^1)$$

Case of  $SL(m, \mathbb{R})$   $P_1 = 0$

We end up with an additional condition;

$$\det(\pi_* U) = \text{trivial bundle.}$$

This is understood in terms of algebraic geometry.

$$\det N_m(\Sigma^1 m, \text{Jac}) = \Sigma^1 m, \text{tr}(\text{Jac}).$$

$$N_m : \text{Jac}(\Sigma^1) \rightarrow \text{Jac}(S).$$

Condition

$$\cup \in N_m^{-1}\{0\} = \text{Prsym variety}$$

is an algebraic variety.

Case of  $PSL(m, \mathbb{R})$

We have an identification on  $SL(m, \mathbb{R})$  bundle;  $PSL(m, \mathbb{R})$  bundles are equivalence classes

$$\rightsquigarrow U \sim V \text{ if } U = V \otimes \pi^* L$$

hence bundles over the spectral curve.

hence the fiber of the Hitchin fibration is identified with

$$\text{Jac}(\Sigma^1) / \pi^* \text{Jac}(S)$$

↪ dual to the Prym variety.

$$N_m^{-1}(0).$$

longlands duals:

maximal torus for  $G$

?

dual maximal torus  $L_G$