LOW DIMENSIONAL STRUCTURES IN DYNAMICAL SYSTEMS WITH VARIABLE TIME LAGS

LIST OF PROBLEMS

(1) **Existence and smoothness of local invariant manifolds at equilibria for neutral differential equations with state-dependent delay**

A particular class of equations is given by

$$x'(t) = A\left(x'\left(t - r(x(t))\right)\right) + f(x(t))$$

with a linear operator A, a delay function r and a smooth nonlinearity f.

(2) **Extension of "lap-number"-techniques to widest possible class of equations with delay**

Interesting classes are for instance the following:

- equations with non-monotonic delay
- equations with implicitly defined delay
- equations with multiple delay

Additionally, in this context numerical case studies could be useful for analytical considerations.

(3) **How does a state-dependent delay change the dynamics of an equation in contrast to a constant delay?**

Here, analytical as well as numerical studies are missing. Interesting situations are for instance

- scalar equation with one delay such as Wright's equation or Mackey-Glass equation
- scalar equation with two or more delays
- higher order equations with one or more delays

(4) **Approximation of delay dependence ($x(t + r(x(t)))$) by distributed/integral terms to improve smoothness properties**

(5) **Global bifurcations in differential equations with state-dependent delay**

Some related issues are

- continuation for homoclinic or heteroclinic solutions
- continuation of (rapidly oscillating) periodic solutions for all relevant parameters

(6) **Analyticity of solutions of analytic delay differential equation (especially, with respect to the delay)**

In particular, the "Whiskey Problem" suggested by Roger

(7) **Study of differential equations with unbounded finite delays as for instance arising in echo control**

Some particular aspects are

- equations with unbounded state-dependent delay
- equation with delay given by an infinite integral

(8) **How can a given ODE dynamics be realized in a class of differential equations with delay?**

(9) **Alternative proofs of the existence of infinite dimensional local invariant manifolds at equilibria for differential equations with state-dependent delay**

For instance, there is no result using the graph transform or Lyapunov-Perron technique to prove the existence of local stable manifolds at equilibria for differential equations with state-dependent delay.