Qualitative Methods in Inverse Scattering

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Approaches to inverse problems

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  - multiple scattering is ignored, hence the problem is linear
  - a priori information is needed

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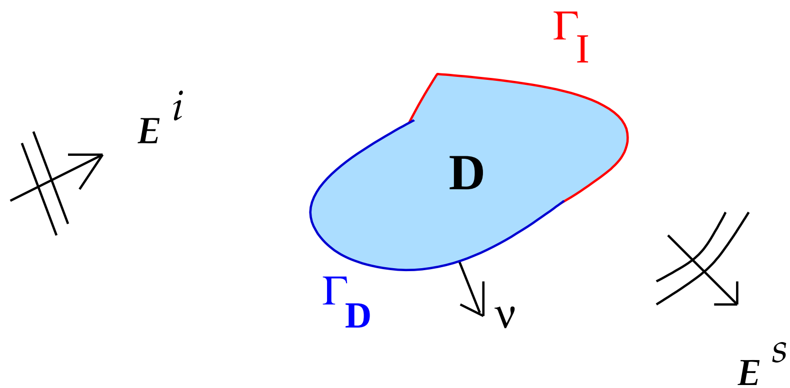
- Optimization techniques:
  - multiple scattering is included, hence the problem is nonlinear
  - a priori information and a good initial guess are needed
  - only one or a few incident waves are needed and the reconstructions are reasonably good
- Qualitative methods:
  - multiple scattering is included however the problem is linear
  - essentially no a priori information is needed
  - multi-static data is needed and only partial information about the scatterer is obtained

Obstacle Scattering Problem



Let $\lambda \in L_\infty(\Gamma_I)$ and positive. The total field u satisfies

$$\Delta_2 u + k^2 u = 0 \qquad \text{in} \qquad \mathbb{R}^2 \setminus \overline{D}$$

$$u = 0 \qquad \text{on} \qquad \Gamma_D$$

$$\frac{\partial u}{\partial \nu} + i\lambda(x)u = 0 \qquad \text{on} \qquad \Gamma_I$$

$$u(x) = e^{ikx\cdot d} + u^s(x)$$

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial r} - iku^s\right) = 0$$

Inverse Scattering Problem

The scattered field u^s has the asymptotic behaviour

$$u^s(x) = \frac{e^{ikr}}{\sqrt{r}} u_\infty(\hat{x}, d) + O\left(r^{-3/2}\right)$$

as $r \to \infty$ where $r = |x|$, $\hat{x} = x/r$, k is fixed and u_∞ is the far field pattern of the scattered field u^s.

The inverse scattering problem is to determine the **shape** D and the **surface impedance** λ, from a knowledge of $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed frequency k, where Ω is the unit circle.

In fact it suffices to know $u_\infty(\hat{x}, d)$ only for $d \in \Omega_1 \subset \Omega$ and $\hat{x} \in \Omega_2 \subset \Omega$

Uniqueness Theorems

Uniqueness of D

Theorem (Kirsch-Kress,1993): D is uniquely determined by $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed value of the wave number k.

Theorem (Liu-Zou, 2007): If D is a polygonal scatterer then D is uniquely determined by $u_\infty(\hat{x}, d)$ for **one** d and $\hat{x} \in \Omega$ and a fixed value of the wave number k.

- *Colton-Sleeman (1983), Gintides (2005)*: finitely many incident waves, Dirichlet boundary condition assuming a restriction on the size of the obstacle
- *Yamamoto, Alessandrini-Rondi, Yamamoto-Elschner*: polygonal domains

Uniqueness Theorems

Uniqueness of λ

Theorem: $\lambda \in C(\overline{\Gamma_I})$ is uniquely determined from $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed value of the wave number k.

The proof can be found in Cakoni-Colton book (2006)

Theorem (Colton-Cakoni-Monk, 2007): $\lambda \in L_\infty(\Gamma_I)$ is uniquely determined from $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed value of the wave number k.

Remark: Since $u_\infty(\hat{x}, d)$ is an analytic function on $\hat{x}$ and d, in the above uniqueness results Ω can everywhere be replaced by a subset $\Omega_0 \subset \Omega$.

Solution of the Inverse Problem

Determination of D

Newton's Method Kress, Hohage, Potthast, Randell, Hettlich, Ganesh, Djellouli ....

We assume $u_\infty(\hat{x}, d)$ is known for $\hat{x} \in \Omega$ and one (or a few) d, and, for sake of simplicity, that ∂D can be represented as

$$x = r(\hat{x})\hat{x}, \qquad \hat{x} \in \Omega.$$

Consider the mapping $\mathcal{F} : r \to u_\infty$ and solve the non-linear and ill-posed equation

$$\mathcal{F}(r) = u_\infty(\cdot,\, d).$$

Newton's Method

Compute the Fréchet derivative $\mathcal{F}'_r$ of $\mathcal{F} : C^2(\Omega) \to L^2(\Omega)$.

- Kirsch, Potthast, Hettlich etc. for Dirichlet/ Neuman boundary condition.
- Haddar-Kress (2004) for impedance condition.
- Bochniak-Cakoni (2004) for mixed Dirichlet-Neumann condition.

The above equation is replaced by the linear equation

$$\mathcal{F}(r) + \mathcal{F}'_r q = u_\infty(\cdot,\, q)$$

which from an initial guess $r = r_0$ yields the new approximation $r_1 = r_0 + q$. Newton's method consists in iterating this procedure.

Partial results on the convergence of the Newton Method are proven by Hohage (1998), Potthast (2001) ....

Qualitative Methods

or Non-iterative methods

- Find the shape, and extract information about the physical properties of the object.
- Rely on little a priori information and do it in a rather quick and simple way.
- Use multistatic data. Shape reconstruction is not very sharp and partial recovery of the physical properties is achieved.

A list of qualitative methods was presented by Potthast in his talk. In particular, the singular source method and range test for the reconstruction of D was described, see Potthast's book (2001).

Our algorithm for solving the inverse scattering problem will be based on the linear sampling method (*Colton-Kirsch 1996*).

The Far Field Operator

Let $\Phi(x,z) := \frac{i}{4}H_0^{(1)}(k|x-z|)$ which has the far field pattern

$$\Phi_\infty(\hat{x},z) = \frac{e^{i\pi/4}}{\sqrt{8\pi k}} e^{-ik\hat{x}\cdot z}.$$

Define the far field operator $F : L^2(\Omega) \to L^2(\Omega)$ by

$$(Fg)(\hat{x}) := \int_\Omega u_\infty(\hat{x},d)g(d)ds(d).$$

The Herglotz wave function with kernel g is defined by

$$v_g(x) := \int_\Omega e^{ikx\cdot d} g(d)ds(d).$$

The Far Field Equation

Define the far field equation

$$(Fg)(\hat{x}) = \Phi_\infty(\hat{x}, z), \qquad g \in L^2(\Omega),\ z \in \mathbb{R}^2$$

Let $z \in D$ and suppose that g solves the far field equation.

Rellich's Lemma $\Longrightarrow u_g{}^s(x) = \Phi(x, z)$ in $\mathbb{R}^2 \setminus \overline{D}$

In particular $-v_g = \Phi(x, z)$ on Γ_D.

As $z \in D \to \partial D$, $\Phi(x, z) \to \infty$ and so does $v_g \Longrightarrow \|g\|_{L^2} \to \infty$.

Solving the Far Field Equation

Unfortunately, in general, the far field equation does not have a solution for any $z \in \mathbb{R}^2$!

For $z \in D$ the far field equation has a solution if and only if the interior boundary value problem

$$\begin{aligned} \Delta v_z + k^2 v_z &= 0 && \text{in} \quad D \\ v_z + \Phi(\cdot,\, z) &= 0 && \text{on} \quad \Gamma_D \\ \frac{\partial(v_z + \Phi(\cdot,\, z))}{\partial \nu} + i\lambda(x)(v_z + \Phi(\cdot,\, z)) &= 0 && \text{on} \quad \Gamma_I \end{aligned}$$

has a solution v_z such that $v_z = v_g$ is a Herglotz function with kernel g.

Approximation by Herglotz function

Define

$$\mathcal{H}(D) := \{u \in H^1(D) : \ \Delta u + k^2 u = 0\}.$$

Theorem (Colton-Sleeman (2001), Colton-Kress (2001)): Suppose that $\mathbb{R}^2 \setminus \overline{D}$ is connected. Then the set of Herglotz functions

$$\{v_g : \ g \in L^2(\Omega)\} \quad \text{is dense in } \mathcal{H}(D).$$

Theorem (Cakoni-Colton (2001)): Suppose that $\mathbb{R}^2 \setminus \overline{D}$ is connected. Then the set of Herglotz functions with kernel supported on a compact subset Ω_0 of the unit sphere Ω

$$\{v_g : \ g \in L^2(\Omega_0)\} \quad \text{is dense in } \mathcal{H}(D).$$

Solving the Far Field Equation

$$(Fg)(\hat{x}) = \Phi_\infty(\hat{x}, z)$$

- For $z \in D$ and a given $\epsilon > 0$ there exists a $g_z^\epsilon \in L^2(\Omega)$ such that

$$\|Fg_z^\epsilon - \Phi_\infty(\cdot, z)\|_{L^2(\Omega)} < \epsilon$$

and the Herglotz wave function $v_{g_z^\epsilon}$ converges in $H^1(D)$ to v_z where v_z is the solution of the interior mixed boundary value problem. Furthermore,

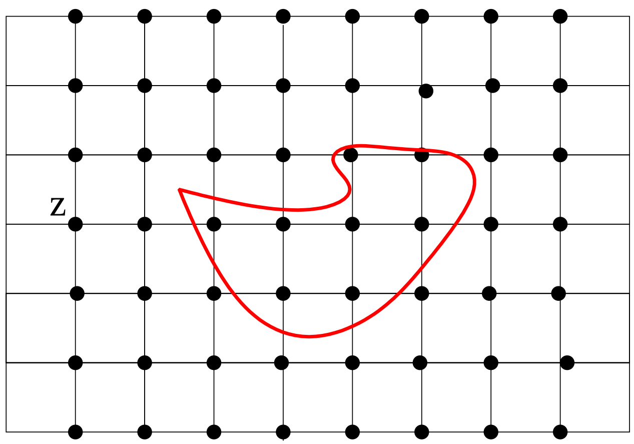
$$\lim_{z \to \partial D} \|v_{g_z^\epsilon}\|_{H^1(D)} = \infty \text{ and } \lim_{z \to \partial D} \|g_z^\epsilon\|_{L^2(\Omega)} = \infty$$

- For $z \in \mathbb{R}^2 \setminus \overline{D}$ and a given $\epsilon > 0$, every $g_z^\epsilon \in L^2(\Omega)$ that satisfies

$$\|Fg_z^\epsilon - \Phi_\infty(\cdot, z)\|_{L^2(\Omega)} < \epsilon \quad \text{is such that} \quad \lim_{\epsilon \to 0} \|v_{g_z^\epsilon}\|_{H^1(D)} = \infty.$$

Indicator Function

Note that for $z \in D$, $v_{g_z^\epsilon}(z) \to v_z(z)$ point-wise.



- Construct a grid $\mathcal{G}$.
- For $z_i \in \mathcal{G}$, solve the regularized far field equation $(\alpha I + F^* F) g_{z_i} = \Phi_\infty(\hat{x}, z)$

The solution of the inverse problem is based on the use of the regularized solution g_{z_i} of the far field equation and $v_{g_{z_i}}$

Open question: Does g_{z_i} and $v_{g_{z_i}}$ behave in the same way as the theoretical g_z^ϵ and $v_{g_z^\epsilon}$?

Factorization Methods

By Kirsch (1998). The far field operator F is replaced by $(FF^*)^{1/4}$. Then, if F is normal (e.g. Dirichlet boundary condition)

$$\Phi_\infty(\cdot, z) \in \text{Range}\,(FF^*)^{1/4} \Longleftrightarrow z \in D.$$

Kirsch, Grinberg etc. has generalized these ideas to the case when F is not normal, replacing $(FF^*)^{1/4}$ by the operator $F_\# := |\text{Re}\,F| + \text{Im}\,F$.

Arens and Lechleiter (2007) have proven when the far field operator is normal then

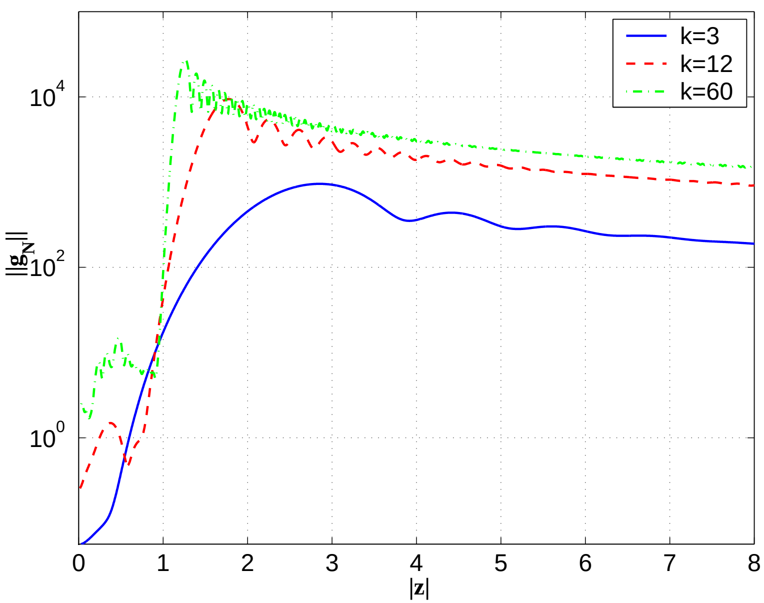
$$2\pi\|\varphi_z\|^2_{L^2(\Omega)} < |v_{g_z}(z)| < 4\pi\|\varphi_z\|^2_{L^2(\Omega)}$$

where g_z is the regularized solution of $Fg_z = \Phi_\infty(\cdot,\, z)$ and φ_z is the solution of $(FF^*)^{1/4}\varphi_z = \Phi_\infty(\cdot,\, z)$.

Determination of D

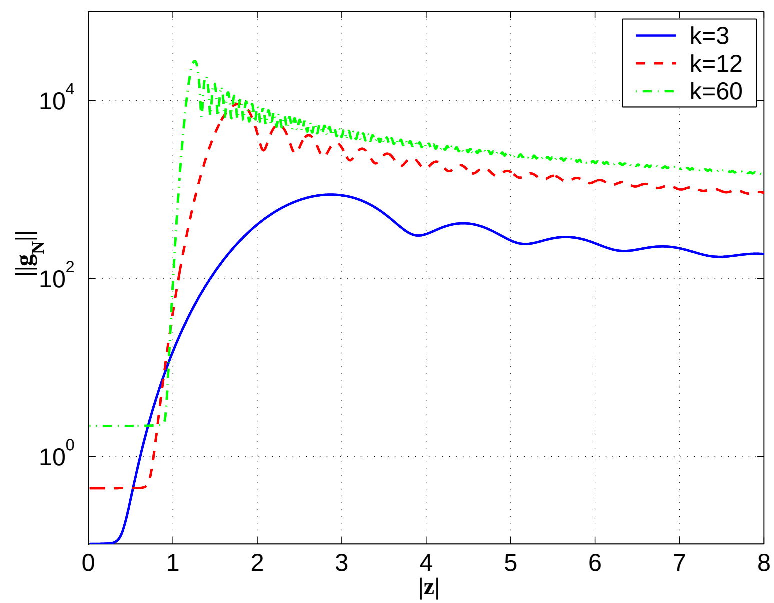
To reconstruct D we plot the level curves $1/||g_{z_i}||_{\ell^2} = c$. The boundary is obtained for an appropriate choice of c.

Collino-Fares-Haddar (2004)



$||g||$ with respect to z:

Dirichlet boundary condition



$||g||$ with respect to z.

Impedance boundary condition

Determination of λ

This is based on the fact that v_{g_z} approximates the solution v_z of interior mixed boundary problem; Cakoni-Colton (2004)

- For every point $z \in D$ we have that

$$\int_{\Gamma_I} \lambda(x) \left| v_z(x) + \Phi(x,z) \right|^2 ds_x = -1/4 - \text{Im}\left(v_z(z)\right).$$

- Let $B_r \subset D$ be a ball of radius r and denote by

$$\mathcal{V} := \left\{ f \in L^2(\partial D_I) : \; f = (v_z + \Phi(\cdot, z))|_{\Gamma_I}, \; z \in B_r \right\}.$$

Then $\mathcal{V}$ is complete in $L^2(\Gamma_I)$

Determination of λ

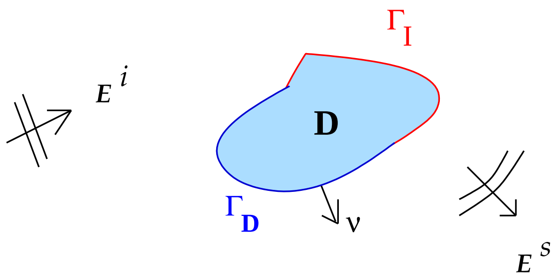
$$\Gamma_D = \text{Support}(v_z)$$

In particular if λ is constant, we obtain

$$\lambda = \frac{-1/4 - \text{Im}\,(v_z(z))}{\|v_z + \Phi(\cdot\,;z)\|^2_{L^2(\partial D)}} \qquad z \in D$$

Recall that v_z is approximated by the Herglotz wave function v_{g_z} where g_z is the regularized solution of the far field equation.

Maxwell's Equations



Let $\lambda \in L_\infty(\Gamma_I)$. The electric total field $E := E^s + E^i$ satisfies

$$\begin{aligned} \nabla \times \nabla \times E - k^2 E &= 0 && \text{in} \quad \mathbb{R}^3 \setminus \overline{D} \\ \nu \times E &= 0 && \text{on} \quad \Gamma_D \\ \nu \times (\nabla \times E) - i\lambda(\nu \times E) \times \nu &= 0 && \text{on} \quad \Gamma_I \\ \lim_{|x| \to \infty} (\nabla \times E^s \times x - ik|x| E^s) &= 0 \end{aligned}$$

where $E^i(x) := ik(d \times p) \times d\, e^{ikx \cdot d}$

Far Field Operator

The scattered field E^s has the asymptotic behaviour

$$E^s(x) = \frac{e^{ik|x|}}{|x|}\left\{E_\infty(\hat{x}) + O\left(\frac{1}{|x|}\right)\right\}$$

as $r = |x| \to \infty$, $\hat{x} = x/r$, k is fixed and $E_\infty(\hat{x}) = E_\infty(\hat{x}, d, p)$ for $\hat{x}, d \in \Omega$ and $p \in \mathbb{R}^3$ is the far field pattern.

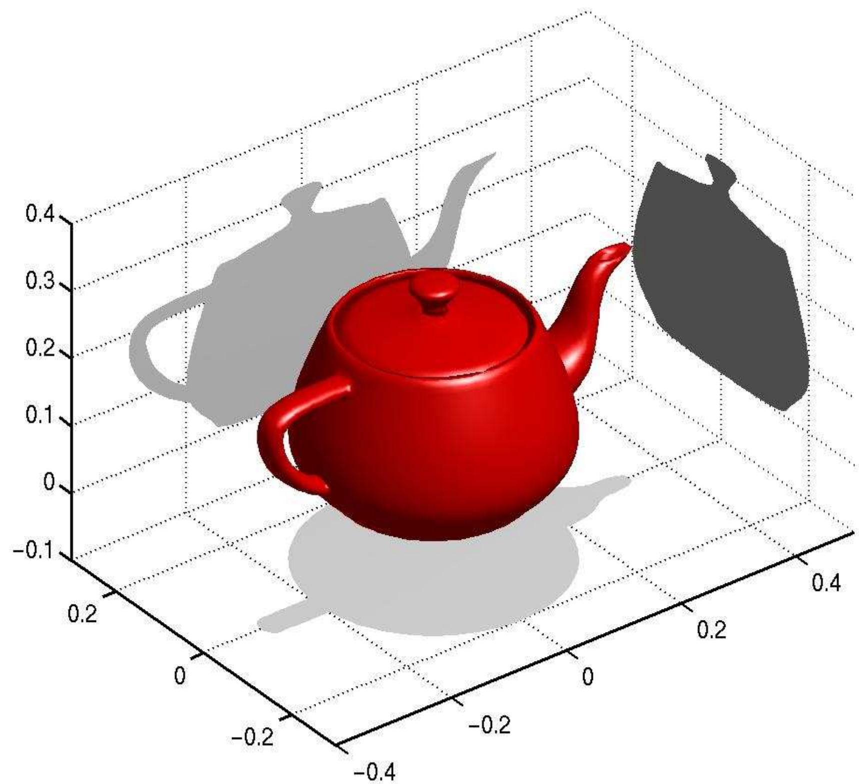
The far field operator $F : L_t^2(\Omega) \to L_t^2(\Omega)$ is defined by

$$(Fg)(\hat{x}) := \int_\Omega E_\infty(\hat{x}, d, g(d))\, ds(d).$$

The far field equation is $(Fg)(\hat{x}) = E_{e,\infty}(\hat{x}, z, q)$.

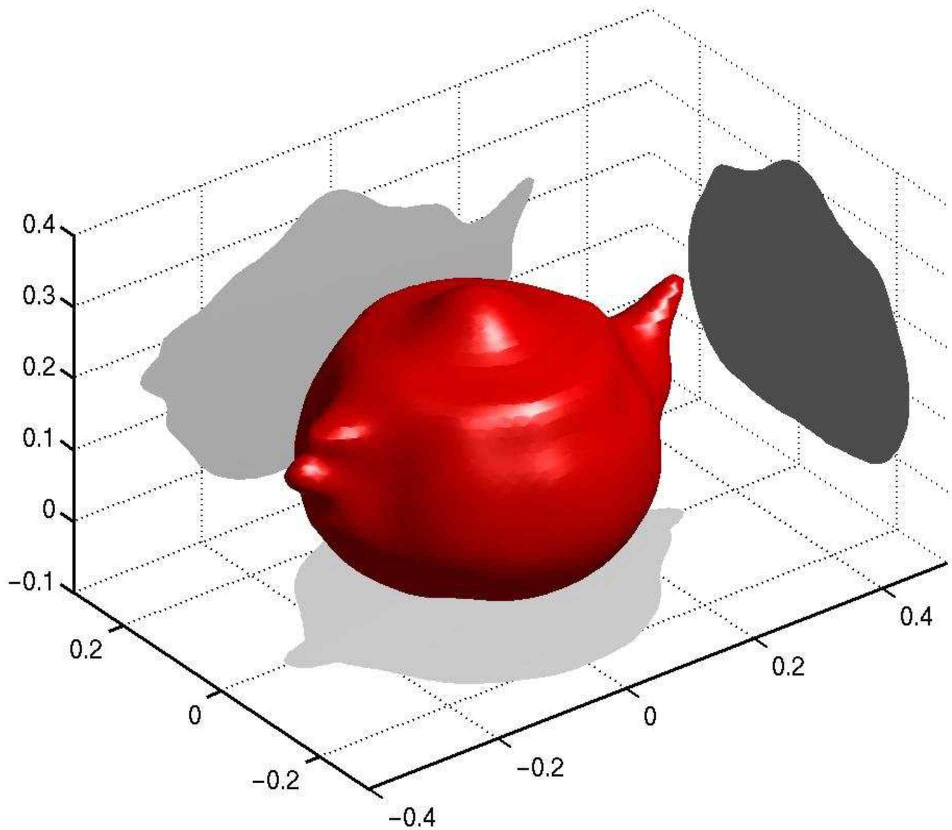
Examples of Reconstruction

Collino-Fares-Haddar (2004)



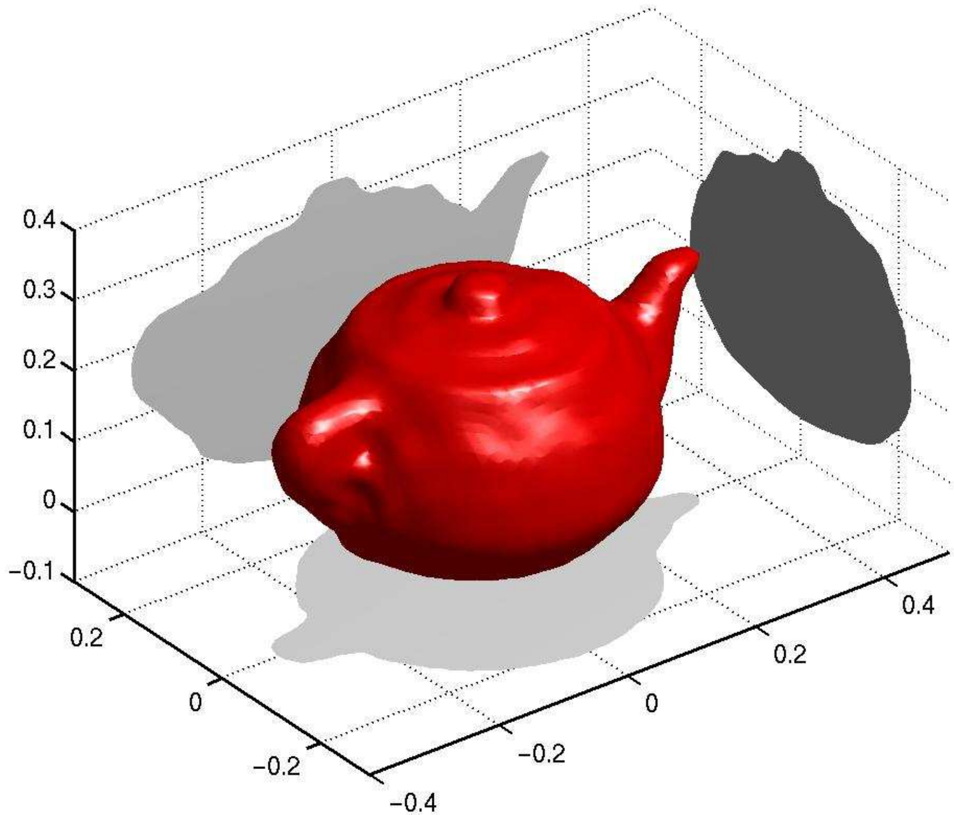
Perfectly conducting teapot, exact geometry

Examples of Reconstruction



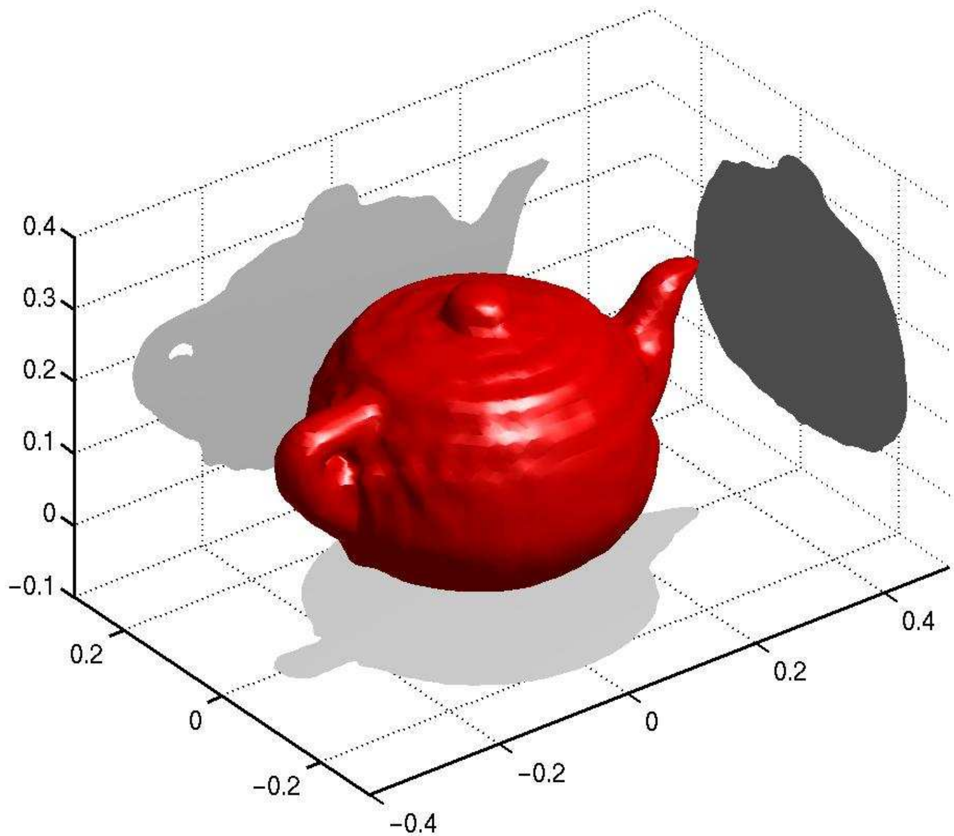
Reconstruction for low frequency

Examples of Reconstruction



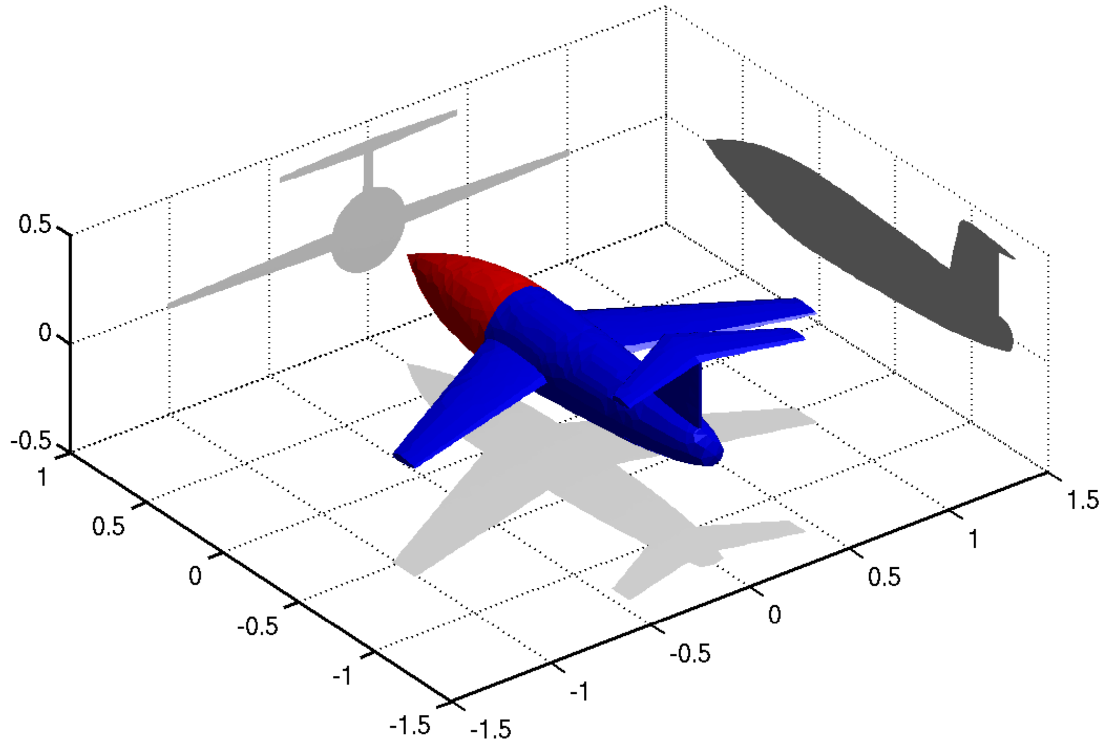
Reconstruction for intermediate frequency

Examples of Reconstruction



Reconstruction for high frequency

Examples of Reconstruction

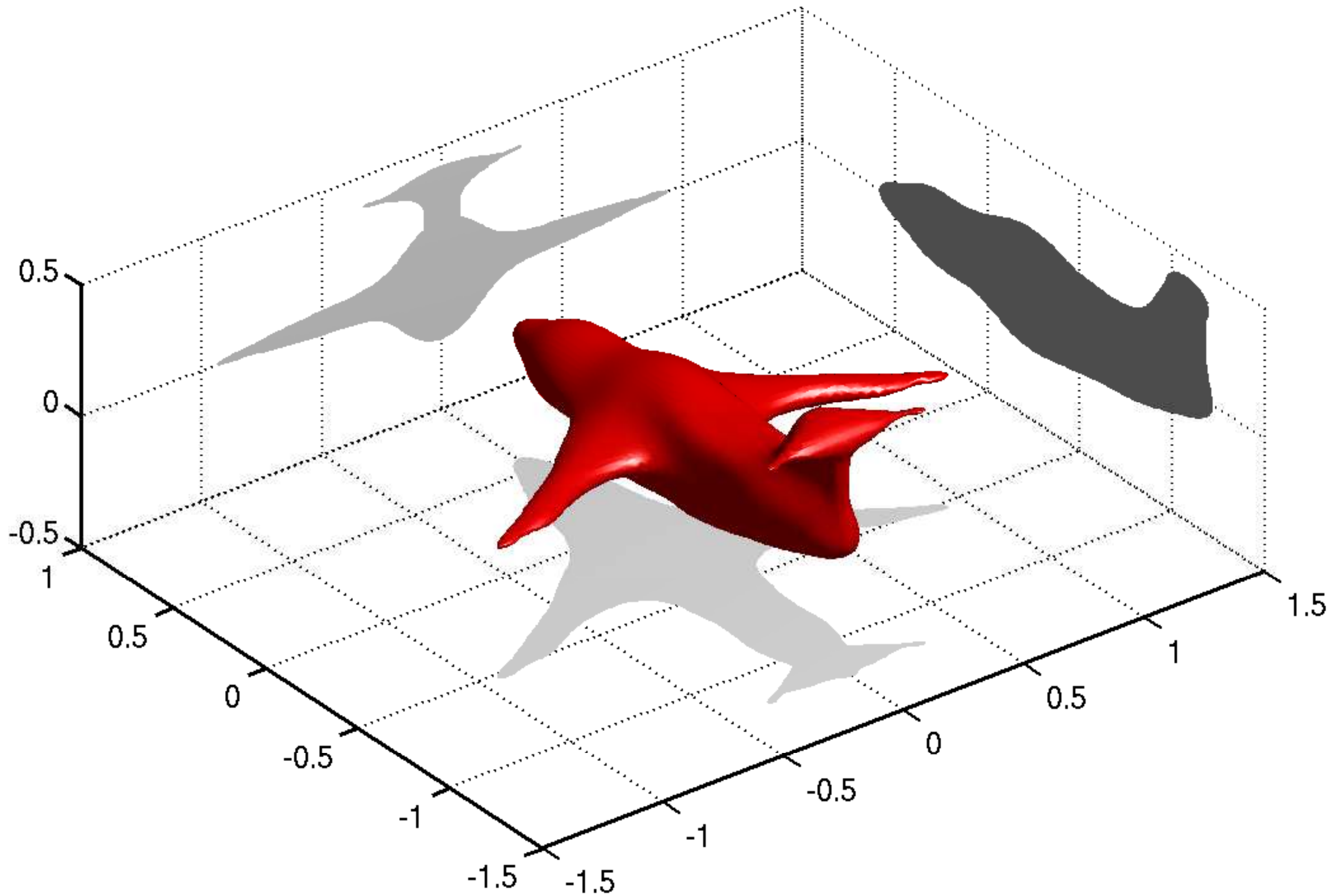


The exact geometry

Impedance boundary condition with $\lambda = 1$ is the red region

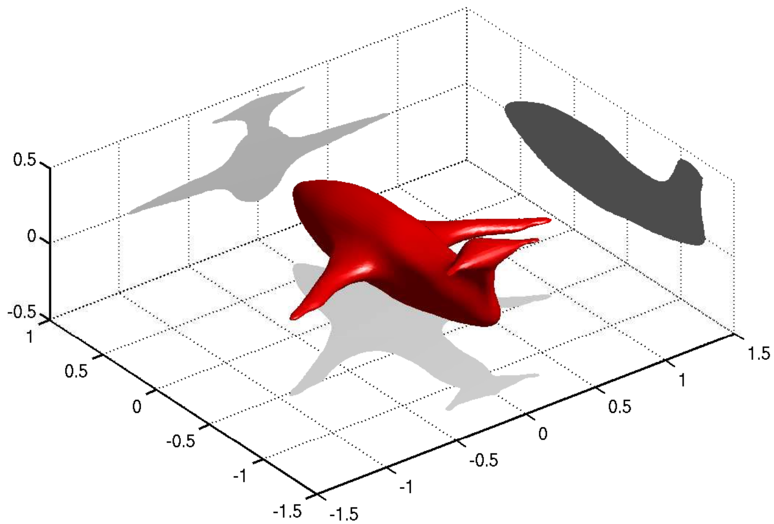
Perfectly conducting boundary condition is the blue region

Examples of Reconstruction

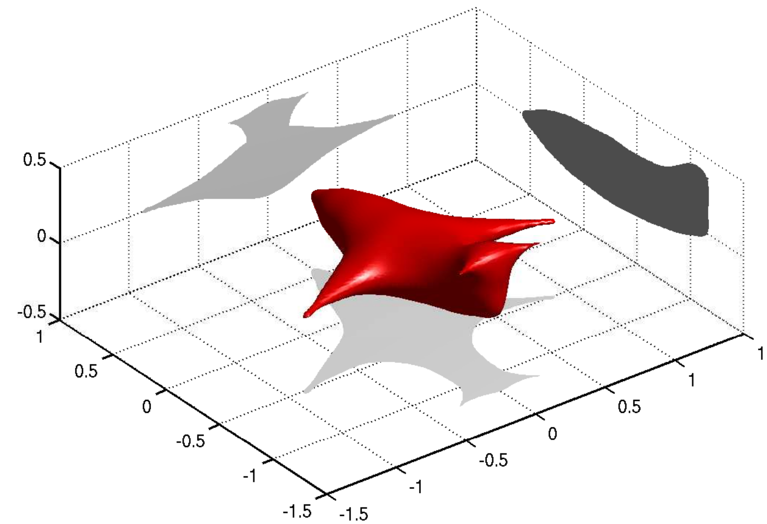


The reconstructed partially coated airplane (wavelength=0.7)

Examples of Reconstruction



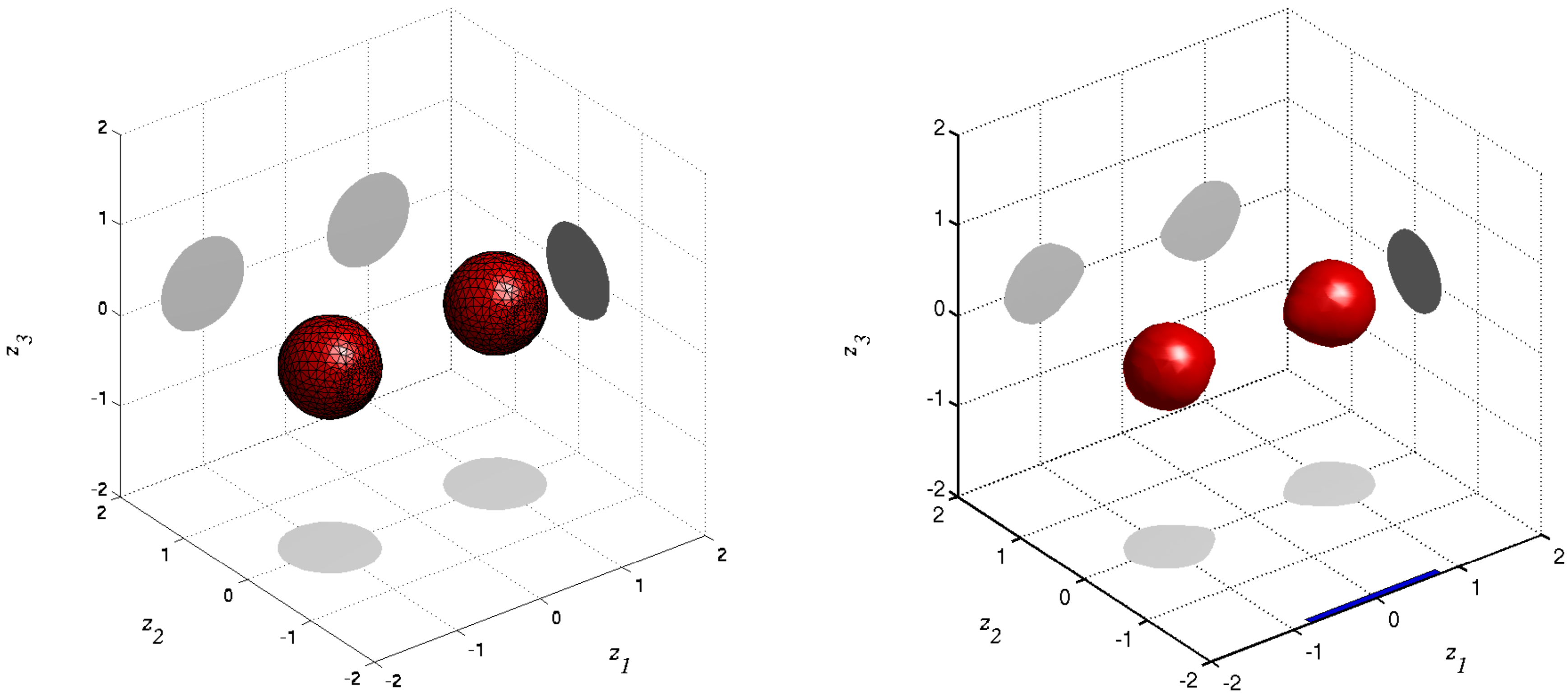
The perfectly conducting airplane



The imperfectly conducting airplane

Examples of Reconstruction

Colton-Monk (2006)



Exact Geometry

Reconstruction

Reconstruction of fully coated two ball with $\lambda = 1$ and $k = 4$.

Examples of Reconstruction

Exact	Exact ∂D	LSM
0.1	0.104	0.11
1	0.99	0.98
1.22	1.21	1.18
2	1.97	1.46

Reconstruction of λ for the two balls. Here $k = 4$.

The Inverse Medium Problem

$$\nabla \cdot A(x)\nabla v + k^2 v = 0 \quad \text{in} \quad D$$

$$\Delta u + k^2 u = 0 \quad \text{in} \quad \mathbb{R}^2 \setminus \overline{D}$$

$$v - u = 0 \quad \text{on} \quad \partial D$$

$$\frac{\partial v}{\partial \nu_A} - \frac{\partial u}{\partial \nu} = 0 \quad \text{on} \quad \partial D$$

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial r} - iku^s\right) = 0$$

$$u(x) = u^s(x) + e^{ikx\cdot d}, \quad d \in \Omega := \{x : |x| = 1\}, \quad \frac{\partial v}{\partial \nu_A} := \nu \cdot A\nabla v$$

A is a symmetric, $\mathcal{R}e\left(\overline{\xi}\cdot A\xi\right) \geq \gamma|\xi|^2$ and $\mathcal{R}e\left(\overline{\xi}\cdot A^{-1}\xi\right) \geq \gamma|\xi|^2$, $\mathcal{I}m\left(\overline{\xi}\cdot A\xi\right) \leq 0$, $\mathcal{I}m\left(\overline{\xi}\cdot A\xi\right) \leq 0$.

Inverse Scattering Problem

The scattered field u^s has the asymptotic behaviour

$$u^s(x) = \frac{e^{ikr}}{\sqrt{r}} u_\infty(\hat{x}, d) + O\left(r^{-3/2}\right)$$

as $r \to \infty$ where $r = |x|$, $\hat{x} = x/r$, k is fixed and u_∞ is the far field pattern of the scattered field u^s.

The inverse scattering problem is to determine the **support** D and the **index of refraction** A, from a knowledge of $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ (and possibly for a range of frequencies k).

Uniqueness Theorems

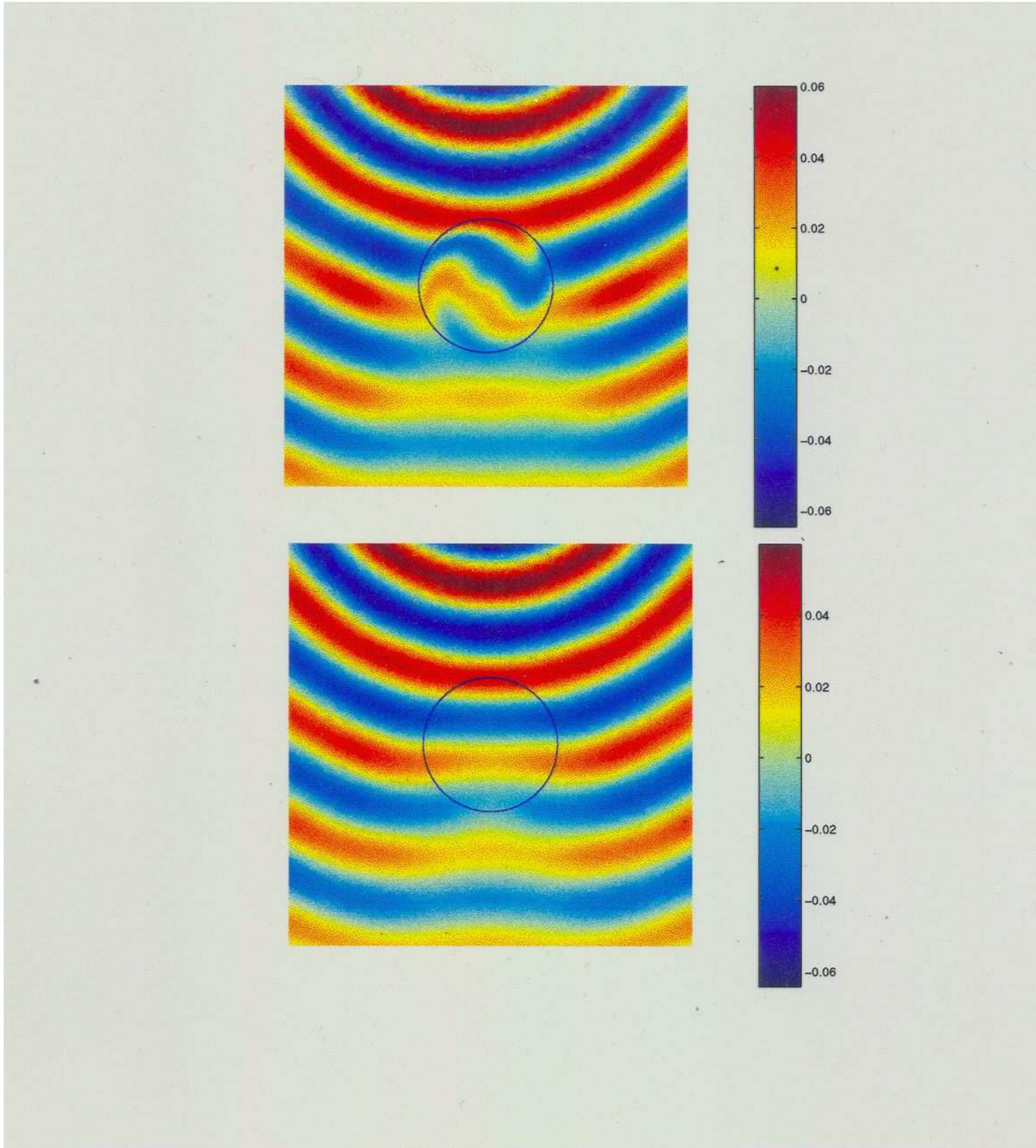
Theorem (Uniqueness of D) Hähner (2000): Assume that either $\bar{\xi}\,\mathcal{R}e\,(A - I)\xi \geq \delta\|\xi\|^2 > 0$ or $\bar{\xi}\,\mathcal{R}e\,(I - A)\xi \geq \delta\|\xi\|^2 > 0$ in D for some δ. Then, D is uniquely determined by $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ and a fixed value of the wave number k.

Uniqueness of A

- Gylys-Colwell (1996); If A is a matrix (anisotropic case) it is known that $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ does not uniquely determine A even if it is known for an interval of values of k.
- If $A(x) = a(x)I$ (isotropic case) Gylys-Colwell (1996) has shown that $u_\infty(\hat{x}, d)$ for $\hat{x}, d \in \Omega$ for a fixed frequency uniquely determine $a(x)$ provided that $a(x)$ is sufficiently close to a constant.

Note that in $\mathbb{R}^3$ the assumption that $a(x)$ is sufficiently close to a constant is not needed; see Nachman (1987).

Non-uniqueness for anisotropic media



The Far Field Equation

$$(Fg)(\hat{x}) := \int_\Omega u_\infty(\hat{x}, d) g(d) ds(d) = \Phi_\infty(\hat{x}, z), \;\; g \in L^2(\Omega), \;\; z \in \mathbb{R}^2$$

For z in D the far field equation has a solution g iff there exist a solution (v_z, w_z) of the interior transmission problem

$$\begin{array}{lll} \nabla \cdot A \nabla w_z + k^2 w_z = 0 \quad \textit{and} \quad \Delta v_z + k^2 v_z = 0 & \textit{in} & D \\ w_z - v_z = \Phi(\cdot, z) & \textit{on} & \partial D \\ \dfrac{\partial w_z}{\partial \nu_A} - \dfrac{\partial v_z}{\partial \nu} = \dfrac{\partial}{\partial \nu} \Phi(\cdot, z) & \textit{on} & \partial D \end{array}$$

such that v_z is a Herglotz wave function $v_g(x) := \int_\Omega e^{ikx \cdot d} g(d) ds(d)$.

ITP: What is known!

Cakoni-Colton-Haddar (2007): Assume that in D for some $\delta > 0$ either $\bar{\xi} \cdot \mathcal{R}e\,(A - I)\xi \geq \delta \|\xi\|^2$ C1 or $\bar{\xi} \cdot \mathcal{R}e\,(I - A)\xi \geq \delta \|\xi\|^2$ C2

- The interior transmission problem satisfies the Fredholm alternative in $H^1(D) \times H^1(D)$.
- If $\mathcal{I}m\left(\bar{\xi} \cdot A\xi\right) < 0$ in D then there are no transmission eigenvalues.
- If $\mathcal{I}m\left(\bar{\xi} \cdot A\xi\right) = 0$ then the set of transmission eigenvalues is discrete.
- Any transmission eigenvalue $k > 0$ must satisfies

$$k^2 \geq \frac{\lambda(D)}{\sup_D \|A^{-1}\|_2} \text{ if C1 holds} \quad \text{and} \quad k^2 > \lambda(D) \text{ if C2 holds,}$$

where $\lambda(D)$ is the first Dirichlet eigenvalue of $-\Delta$ on D.

Remarks

Definition: The values of k for which the homogeneous interior transmission problem (i.e. $\Phi(\cdot,\ z) = 0$) has a non trivial solution are called transmission eigenvalues.
Open Problem Do transmission eigenvalues exists?

The first result in this direction is due to Sylvester and Päivärinta (2007) for the case of $\Delta u + k^2 n(x) u = 0$. They have shown that transmission eigenvalues exist provided the contrast $n - 1 > 0$ is large enough.

Remark: If k is a transmission eigenvalue and v_z is a Herglotz wave function then the far field operator $F : L^2(\Omega) \rightarrow L^2(\Omega)$ is not injective with dense range, where v_z, w_z is the non zero solution of the homogeneous interior transmission problem, i.e. for $\Phi(\cdot, z) = 0$.

Solution of the Inverse Problem

The support D can be determined by the linear sampling method.

What, if anything, can be said about A from a knowledge of u_∞?

Recall two results we have proven: Assume that $\mathcal{I}m\left(\overline{\xi} \cdot A\xi\right) = 0$

- Any transmission eigenvalue $k > 0$ must satisfies

$$k^2 \geq \frac{\lambda(D)}{\sup_D \|A^{-1}\|_2} \qquad \text{if} \qquad \|\mathcal{R}e\, A^{-1}\|_2 \geq \delta > 1$$

  where $\lambda(D)$ is the first eigenvalue of $-\Delta$ on D.

- If k is a transmission eigenvalue and v_z is a Herglotz wave function then the far field operator $F : L^2(\Omega) \to L^2(\Omega)$ is not injective with dense range.

Estimates for A

The **first result** provides an estimate for the 2-norm of A

- Assume that $\|A^{-1}(x)\|_2 \geq \delta > 1$ for all $x \in D$ and some constant δ. Then,

$$\sup_D \|A^{-1}\|_2 \geq \frac{\lambda(D)}{k^2}$$

where k is the first transmission eigenvalue and $\lambda(D)$ is the first eigenvalue of $-\Delta$ on D.

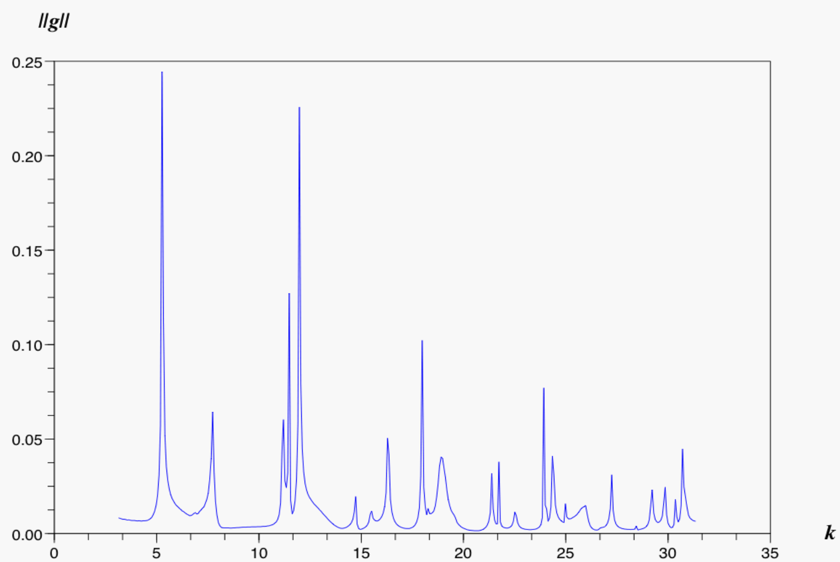
The **second result** provides a way to compute the first transmission eigenvalue from the far field. In particular, the norm of the (regularized) solution to

$$(Fg)(\hat{x}) = \Phi_\infty(\hat{x}, z_0) \qquad z_0 \in D$$

should be large for such value of k; Cakoni-Colton-Haddar (2007).

Transmission Eigenvalues

D is the L-shape $= \{[-0.5,\ 0.5] \times [-0.5,\ 0.5]\} \setminus \{]0,\ 0.5]\times]0,\ 0.5]\}$, $A^{-1} = nI$, $n = 4$ and $\eta = 0$.

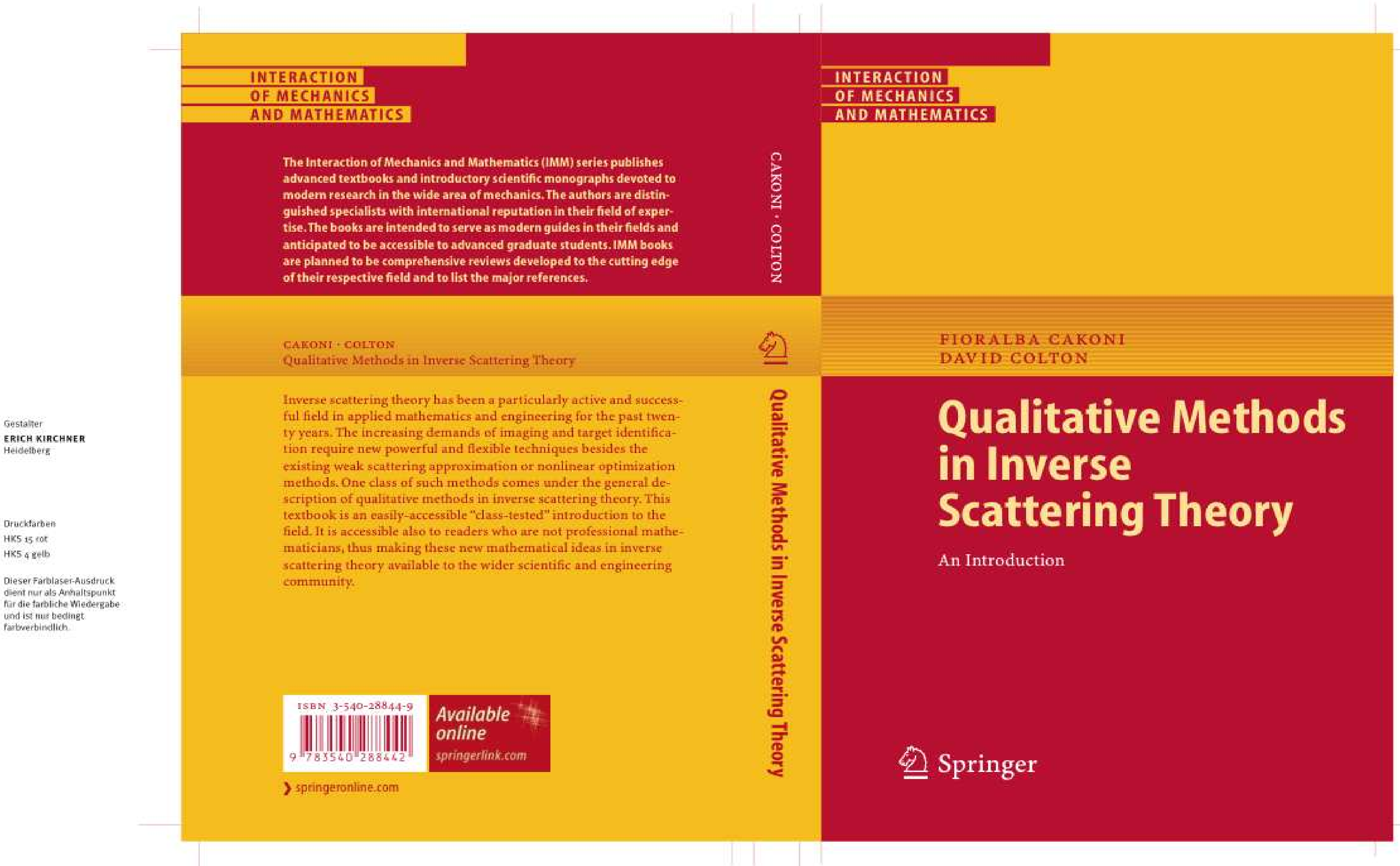


Numerical Examples

D is the L-shape $= \{[-0.5,\ 0.5] \times [-0.5,\ 0.5]\} \setminus \{]0,\ 0.5] \times]0,\ 0.5]\}$,
$A^{-1} = nI$, $\eta = 0$ and $\lambda(D) = 38.6$

n	2.	3.	4.	6.	9.	12.	16.
k_0	15.5	8.1	6.3	4.5	3.3	2.8	2.3
$n_{\min}$	0.2	0.6	1.	1.9	3.5	4.9	7.2

First transmission eigenvalues (k_0)
and lower bounds of the index of refraction $A^{-1} = nI$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