Rational curves and A1-homotopy theory

organized by
Aravind Asok and Jason Starr

Workshop Summary

This workshop brought together people working around the theory of rationally connected varieties (in particular, aspects related to existence and density of rational points, and higher notions of rational connectivity), and people working in A^1-homotopy theory and related "motivic" aspects of algebraic geometry. Introductory lectures providing some common ground for the participants were given by Hassett (rational connectivity), Levine (A^1-homotopy theory), Haesemeyer (splitting varieties) and Colliot-Thélène (obstructions for rational points). Research talks were given later in the week by Esnault, Voisin, Krashen, Xu, DeLand, and Doran. Subsequent discussion focused on cohomological aspects of rationally connected varieties, obstructions for existence of rational points (especially using ideas from obstruction theory), and geometry of splitting varieties (generic and p-generic splitting varieties, and various generalizations).

- Obstructions to the existence of rational points: The participants tried to use the A^1-fundamental (sheaf of) group(s) to recover old and produce new obstructions to existence of rational points. Non-abelienness and lack of base-point produces complications, but new ideas were discussed and some concrete questions for G-torsors were raised.
- In order to study the obstructions suggested in the previous point, further computations of the A^1-fundamental group are required. "Generators and relations" presentations of the A^1- fundamental group were discussed. Known computations for rational surfaces, toric varieties and closely related "cellular" varieties were mentioned, with the aim of computing some obstructions.
- It is known that rationally connected varieties have no non-trivial pluri-differential forms. The converse was investigated. In particular, it was asked whether one could prove directly that a variety without pluri-differential forms had a trivial fundamental group. This problem was studied in the context of the Shafarevich conjecture, and also related to some work of Gromov on varieties with "large" fundamental group.
- The definition of rationally simply connected varieties was reviewed. In particular, the question "which geometric and arithmetic properties should characterize rational simple connectedness?" was discussed in detail. Examples of such properties include existence of points over function fields of surfaces, weak approximation, etc.
- The birational geometry of norm varieties and some generalizations was discussed. New kinds of splitting varieties for cohomological operations were constructed. For example, generalizing the notion of a norm variety, participants studied the problem of existence of varieties having a rational point if and only if a Massey product in étale cohomology exists. Such varieties exist and can be realized rather explicitly in terms of bundles of Brauer-Severi varieties. Wickelgren will investigate this further.

- Colliot-Thélène and Voisin observed that a birational invariant studied by Voisin was the same as an alternative invariant studied by Colliot-Thélène and Ojanguren.
- The Russell cubic threefold was investigated in detail. In particular, the A^1-homotopy theory of this variety (e.g., its A^1- connectedness properties) and its motive were studied.