AIM: ARITHMETIC STATISTICS OVER FINITE FIELDS PROBLEM SESSION

Question 1 (Brian Conrey)**.** Let $E/\mathbb{F}_q(T)$. Let f vary over square free, large degree n and let $P \in \mathbb{P}^1$; does

$$\lim_{n\to\infty} \frac{\#\{f \in \mathbb{F}_q[T]_n : \operatorname{rank} E_f \geq 2 \text{ and } f(P) \in (\mathbb{F}_q^*)^2 - \{0\}\}}{\#\{f \in \mathbb{F}_q[T]_n : \operatorname{rank} E_f \geq 2 \text{ and } f(P) \notin (\mathbb{F}_q^*)^2\}} =^? \sqrt{\frac{\#E_P(\mathbb{F}_q)}{\#\widetilde{E}_P(\mathbb{F}_q)}}$$

(This is conjectured for number fields based on numerical evidence, random matrix theory, moments of L-functions, and BSD.)

Question 2 (Melanie Matchett Wood)**.** Develop a version of the BKLPR heuristics with $\mathbb{Z}_p$ replaced by $\mathbb{Z}$, counting up to height B. Use the rank 2 case to calibrate B with the height used when counting elliptic curves. Then use that calibration to make a rank 3 prediction and compare this with predictions coming from random matrix theory.

Question 3 (Jordan Ellenberg)**.** Understand which ways of counting field extensions satisfy Principles (A) and (B).

Question 4. Which definition(s) of automorphism groups give rise to the nicest formulas?

Question 5. For cyclic triple covers of $\mathbb{P}^1$ can we recover (A) and (B) while counting by genus?

Question 6 (Alina Bucur, Kiran Kedlaya)**.** Are principles (A) and (B) easier/harder/true after conditioning on ramification information?

Question 7. Does (A) fail for $\mathbb{Z}/4\mathbb{Z}$ function field extensions, counted by genus?

Question 8 (Kiran Kedlaya)**.** Fix q. Describe (conjecturally) the distribution of the number of $\mathbb{F}_q$ points on curves of genus g as g tends to ∞.

Question 9 (Kiran Kedlaya)**.** Look at principle (A) for tri-canonically embedded curves.

Question 10 (Melanie Matchett Wood)**.** Look at complete intersections, and some examples with a moving target space.

Question 11 (John Voight)**.** When do counts get nicer if we are willing to partially compactify?

Question 12 (Julio Cesar Bueno de Andrade)**.** Fix q, $h(x) \in \mathbb{F}_q[x]$, $k \geq 3$. Get asymptotics for

$$\sum_f d_k(f) \cdot d_k(f+h)$$

where f ranges over degree n polynomials, as n tends to infinity.

Question 13 (Pollack)**.** What can we say (resp. what do we know) about gaps between irreducible polynomials over $\mathbb{F}_q$?

Question 14 (Pollack)**.** Over a function field, how many coefficients of a polynomial (and in which positions) can be specified without eliminating the possibility of irreducibility?

Question 15 (Jordan Ellenberg)**.** What algebro-geometric facts about elliptic surfaces explain Poonen-Rains/BKLPR?

Question 16 (Brian Conrey)**.** Fix q, let $g \to \infty$, and describe the distribution of discriminants of L-polynomials of curves (or hyperelliptic curves).

Question 17 (John Voight)**.** What is the average number of points on the Jacobian of a hyperelliptic curve?

Question 18. Fix q. How many different zeta functions are attached to curves of genus $g \gg q$?

Question 19 (Kiran Kedlaya)**.** Fix g, fix C/K a curve of genus g, where $K = \mathbb{F}_q(X)$ and X is not necessarily rational. Can we compute the monodromy group of C? I.e., can we compute

$$\{\overline{\operatorname{im}(\operatorname{Gal}(K) \to \operatorname{Aut}(H^1(C, \overline{\mathbb{Q}_p})))} : C \in M_g(K)\}$$

Question 20 (Melanie Matchett Wood)**.** When do (A) and (B) predict an interesting average value of $\#C(\mathbb{F}_q)$?

Question 21 (Jordan Ellenberg)**.** In what ways are discriminants of random trigonal curves like random polynomials?

Question 22 (Katz)**.** The Heilbron sums

$$\sum_{x \mod p} \exp \frac{2\pi i x^p(1+pt)}{p^2} = \sum_{x \mod p} \exp \frac{2\pi i x^p}{p^2} \cdot \exp \frac{2\pi i p t}{p^2}$$

seem to approximate (as t varies) the (push forward of) Haar measure better than expected. Why?

Question 23 (Jordan Ellenberg)**.** Describe the right hand side of variance of $\Lambda(n) \sim q^{?} \cdot n$ (as $q \to \infty$) geometrically.

Question 24 (Brian Conrey)**.** What is the variance of

$$\sum_f \Lambda(f)\Lambda(f+h)$$

(considered as a function of h).

Question 25 (Brian Conrey)**.** Carry out (the other) Levinson's method for function fields.

Question 26 (Brian Conrey)**.** Do the analogue of Nymon-Barling criterion for RH over function fields (leading to a different proof of RH).

Question 27 (1st Tue speaker)**.** Ramanujan sums

$$c_q(x) := \sum_{a<q,(a,q)=1} \exp 2\pi i(a/q)x$$

can write Λ in terms of $c_q(X)$ and other arithmetic functions too. How do these look in the function field setting?

Question 28 (Jordan Ellenbird)**.** What is the role of wild ramification in questions a la Melanie Matchett Wood's work. (E.g. hyperelliptic curves in characteristic 2.)

Question 29 (Chantal Davis)**.** Local statistics for zeros for L-functions of orthogonal type.

Question 30 (Rubenstein)**.** Can we compute averages of the Hardy-Littlewood constants in the function field setting look like? Yes.