## AUTOMORPHIC KERNEL FUNCTIONS WORKSHOP

These problems were stated in the problem brainstorming session:

## Basic Functions:

- (1) What does one need to know about "basic functions" to put them in the trace formula? What are asymptotics of the "basic functions"?
- (2) Define and explain the Fourier transform for Vinberg monoids. Find their relations to the Langlands-Shahidi method and the Rankin-Selberg method.

## Endoscopy and Beyond:

- (1) Can beyond endoscopy be used to prove that the  $R(\theta)$ -bounds in the work of Jiang-Liu are sharp?
- (2) Relative endoscopy: Develop the theory of relative endoscopy in conjunction with the relative trace formula.
- (3) Referring to MR3117742: there are transfer factors from an elliptic torus in  $GL_2$  to  $GL_2$ . What about transfer factors from  $GL_2$  to an elliptic torus?
- (4) What are the relations between the intertwining operators and character relations? See Labesse-Langlands.

## **Beyond Endoscopy:**

- (1) Isolate contributions of the continuous "special" representations on the geometric side of the trace formula. Here, *special* refers to larger poles of relative *L*-functions. Some special cases may be related to the Rankin-Selberg integral method, for instance, the doubling integral method of Piatetski-Shapiro and Rallis on  $GL_n$  or  $GL_2$ .
- (2) Take the current literature on beyond endoscopy, and make a list of the precise local and global statements required. For example: local matching and global matching.
- (3) Arthur has introduced the so-called *r*-trace formula. Describe (and name!) the *r*-"beyond endoscopy" groups and discuss the meaning of geometric matching.
- (4) How does the functional equation give an expression for the *r*-trace formula? In other words, how does the *r*-Fourier transform play roles here?
- (5) Isolate the contribution of hyperkloosterman sums to the Kuznetsov trace formula and explain it spectrally.