New p-adic perspectives on canonical integral models for Shimura varieties

organized by
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Workshop Summary

Introduction

The purpose of this workshop was to bring together experts from around the globe to discuss and study problems arising in the theory of integral canonical models of Shimura varieties.

In recent years, there has been tremendous progress on various parts of the program, starting with the seminal work of Kisin in the 2010s. More recently, work of Pappas–Rapoport has brought perfectoid methods from the work of Scholze–Weinstein and Fargues–Scholze to formulate a characterization of integral models in these terms. In the last couple of years, we have also had inputs from prismatic methods developed by Bhatt–Scholze, Bhatt–Lurie and Drinfeld, which have led to a better understanding of the theory at least for integral models at places where the level is hyperspecial. Finally, also in the past two years, the work of Bakker–Shankar–Tsimerman has expanded the world of integral models to include Shimura data of exceptional type, which do not have any connection whatsoever to the moduli of abelian varieties.

The intent of the organizers was to bring all these new developments to a wider (and younger) audience, and to foster open and generous sharing of ideas for current outstanding problems in the area, as well as to engage in some more speculative ventures with an eye towards future developments. One concrete goal was to better understand integral models at places of parahoric, but possibly non-hyperspecial level.

With AIM's wonderful support, we think these goals were more than met, and we believe that there will be a substantial scientific contribution to the field as a consequence of the conversations and discourse arising directly (and indirectly) from the workshop.

Morning talks

The morning talks were of two different flavors: At the beginning of the week, we had introductory ones by the organizers and others: topics covered included the Pappas–Rapoport characterization of integral models, the approach of Madapusi–Youcis to the construction and characterization of models at hyperspecial level, applications of the theory of integral canonical models to the semisimplicity of Frobenius, and the theory of Igusa stacks. Towards the latter half, we had a mix: We had talks by some of the younger participants, including new postdocs and graduate students, giving them an opportunity to share their work with the other attendees. We also had talks from a couple of the more senior attendees, including

Michael Rapoport and Xinwen Zhu, who told us about some quite exciting new developments in the story, going beyond parahoric levels.

Problem groups

We started on Monday with 20 posed problems. The organizers selected eleven (11) of these for voting on Tuesday, and by the end of the week, four of them ended up with a substantial base of interest among the attendees. All of these problems have a very novel flavor, and progress in any of them will be of great scientific interest.

The Oort–Tate group scheme.

This group studied the perfect F-gauges attached to finite flat group schemes coming from Oort–Tate theory.

Oort and Tate constructed a universal finite flat commutative group scheme G of prime order p over a certain Artin stack OT. Moreover, they described such group schemes in terms of the data of a line bundle L together with two homomorphisms $L \to L^{\otimes p}$ and $L^{\otimes p} \to L$. On the other hand, Madapusi–Mondal recently identified the category of finite flat p-power torsion commutative group schemes over a p-adic formal scheme with a full subcategory of perfect prismatic F-gauges on that formal scheme.

Our first goal was to gain a better understanding of the F-gauge on the p-completion of OT corresponding to the universal group scheme. We studied its structure after restriction to the formal deformation ring of a supersingular elliptic curve at level $\Gamma_0(p)$ and explored how to give a more global description. Our second goal was to find a procedure that describes L and the two homomorphisms directly in terms of the F-gauge, instead of going through the finite group scheme. We thought about what happens when one restricts to a perfect $\mathbf{F}_p$-algebra, where everything becomes more explicit. More generally, we were able to directly recover the two homomorphisms, considered as two-term perfect complexes.

In the future, we hope to expand on our work toward the second goal and describe the line bundle in terms of the perfect F-gauge for any p-adic formal scheme, with potential applications to the integral models for Shimura varieties at $\Gamma_1(p)$-type level of Pappas–Rapoport.

Cyclic base-change.

This group identified the precise statements about (global) Shimura varieties that would be necessary for the application to showing a cyclic base change compatibility result in the setting of the Fargues-Scholze local Langlands correspondence. In particular, a more precise statement than what is implied by Bakker–Shankar–Tsimerman is needed, though only in the setting of compact Shimura varieties. We also discussed some strategies for proving these more precise statements; there are a few plausible approaches, and we will continue to work this out.

Exotic Hecke correspondences.

The group start with understanding exotic hecke correspondences (constructed by Xinwen Zhu and Liang Xiao) in some PEL situations. Xinwen also explained the real group theory behind why and when these exotic hecke correspondences should exist, and introduced to us the archimedian analogue of $B(G, \mu)$. For some of the time, the group focussed

on the specific example of two GL_2-type Shimura varieties, one a Hilbert modular threefold and the other a Shimura curve of Mumford type, where such correspondences ought to exist but haven't yet been constructed. The group reduced constructing them to showing that a certain type of local system should underlie an abelian scheme.

Dilatations.

This group considered the problem of the right replacement for the stack of apertures when the reductive group scheme $\mathcal{G}$ is replaced by a a parahoric Bruhat-Tits scheme which appears as the dilatation along a parabolic subgroup of the mod-p fiber. This would include as a primary example the case of Iwahori level. A first suggestion was to try and develop an equivalent construction by using the new theory of sheared displays over sheared Witt vectors, as explained in work of Drinfeld and Hoff–Lau. Another suggestion was to examine the constructions arising in the work of Lee–Madapusi on spaces of isogenies. This hints about the right desiderata one should have in order to set up the theory correctly, and we realized that ideas of George Pappas from prior work on local models can be reinterpreted in a unified framework, where the local model arises quite naturally from the geometry of the situation.