Classification theory for abstract elementary classes

organized by
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Workshop Summary

Saharon Shelah, in his recently published list of open problems in model theory [Sh 702], writes, "I see this [classification of Abstract Elementary Classes] as the major problem of model theory." Shelah in the mid seventies proposed a categoricity conjecture as an easy to state but very difficult test problem. Shelah alone published many hundreds of pages dedicated to partial solutions and generalizations of his conjecture. Despite much progress in recent years, a complete solution of the categoricity conjecture is still far away.

In recent years Grossberg and VanDieren isolated the notion of tameness as a natural assumption to develop classification theory for Abstract Elementary Classes (AECs). They also proved an instance of Shelah's categoricity conjecture for tame AECs. This work influenced several people (among them Tapani Hyttinen, Meeri Kesälä and Olivier Lessmann) to obtain further results, from stronger assumptions.

In parallel Boris Zilber discovered that methods of AECs (developed by Shelah more than 25 years ago) can be used to give some illumination on why Schanuel's conjecture from transcendental number theory might be true. The conjecture implies (among many stronger results) that $\mathbf{e} + \pi$ is a transcendental number.

As we did not think that it is likely to make progress on the main categoricity conjecture within a short workshop, we decided to accomplish two more modest goals: share some of the main, recent results of the pure theory with a diverse body of people and try to understand some of the abstract results in concrete algebraic context, setting the ground for potential applications of the subject in algebra and geometry. To accomplish this we invited several generations of model theorists who work in pure and applied (mainly algebraic and number theoretic) aspects and a high percentage of recent PhDs and graduate students. We identified several potential speakers and ahead of time asked them to prepare informal presentations. The speakers were told that their talks would be incorporated into the workshop as the workshop took place. During the week of the workshop, the talks were altered and fine-tuned to reflect participant interests and progress. Only the first two, background talks in the first day were pre-planned.

During the first two days of the workshop, we had productive sessions to discuss some of the big problems of the field as well as problems that should be dealt with during the workshop. Several working groups were formed addressing (1) Fields, (2) Groups, (3) Finitary AECs, (4) Dimension theory in AECs and (5) Refinement of Shelah's presentation theorem. As the workshop progressed some groups naturally dissolved, and new groups formed. Only working groups (1), (2) and (5) continued to the very end.

Group (2) proposed by Wesely Calvert initiated the translation of concepts of AECs into appropriate classes of Abelian groups. They managed to identify that some natural classes of Abelian groups are AECs and they were also able to compute where exactly in the stability hierarchy (of AECs) these examples fit. This work, in addition to providing new algebraic examples that occur naturally in mathematics (unlike earlier combinatorial examples), might be of importance in the development of geometric model theory for AECs.

Working group (1) was proposed by Tom Scanlon to classify $\aleph_0$-stable AECs of fields. By Macintyre's theorem when the class has an axiomatization as a complete first-order theory then the fields must be algebraically closed. The results of this group were the most surprising. It was established that there are classes of fields which are AECs, satisfying the amalgamation property, having arbitrary large models which are categorical in all uncountable cardinals (hence $\aleph_0$-stable), but unlike in first-order model theory are not algebraically closed. In fact it seems that many of the stability classes can be coded into a nicely behaved AEC of fields. It was conjectured that even the Hart-Shelah example can be coded into a nicely behaved AEC of fields (with the same spectrum function). Several new problems were discussed: whether there are geometries (including non-first-order ones) that occur already in first-order model theory, and whether (like for simple first-order theories) there is a uniqueness theorem of dependence relations in AECs.

Maryanthe Malliaris proposed that the fifth working group investigate whether or not there exist specializations of Shelah's Presentation Theorem that could shed light onto the relative generality of various AECs such as homogeneous, finitary and tame. This working group explored several avenues and if nothing else gained incredible insight into the delicacy of Shelah's proof of the presentation theorem.

The workshop achieved more than we expected. Several central and feasible problems were identified. Not only were many new and interesting algebraic examples identified, but also significant differences between algebraic aspects of model theory of AECs and first-order were discovered. A number of the participants have continued the collaborations started at AIM on problems discussed in the problem sessions, especially in: (1), (2) and (5).