1. Questions

1.1. Computable approximability (Calvert)

Definition. \( \mathcal{M} \) is computably approximable if for every computable \( \mathcal{L}_{\omega_1, \omega} \) sentence \( \varphi \) true of \( \mathcal{M} \) there is a computable \( \mathcal{N} \models \varphi \) such that \( \text{SR}(\mathcal{N}) < \omega_1^{\text{CK}} \), where \( \text{SR} \) denotes Scott rank.

Question 1. Is it the case that every computable structure is computable approximable?

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**Question 2.** Let $\varphi \in L_{\omega_1,\omega}$ be a satisfiable sentence of quantifier rank $\alpha$, and suppose that either $\omega_1 > \beta > \alpha$ or $\beta > \alpha + \omega$. Is there an $\mathcal{M} \models \varphi$ such that $\text{SR}(\mathcal{M}) < \beta$?

1.2. **Strongly minimal nontrivial locally modular nonorthogonal groups (Medvedev).**
Let $\mathcal{M}$ be a strongly minimal nontrivial locally modular structure. There is a nonorthogonal interpretable strongly minimal group $G$.

**Question 3.** How difficult is it to find a presentation of $G$ in terms of $\mathcal{M}$?

**Question 4.** How difficult is it to find a presentation of $\mathcal{M}$ in terms of $G$?

Consider a three-to-one map $(\mathbb{Q}, +) \leftarrow (\mathcal{M}, \oplus)$ where $(\mathcal{M}, \oplus)$ is strongly minimal.

**Question 5.** Must $\oplus$ be definable in some $\mathbb{Q} \times F$, where $F$ is a finite set?

1.3. **Continuous sections (Miller).**
Consider $T$ stable. Then for all $\mathcal{M} \prec \mathcal{N} \models T$, the map $S_1(\mathcal{N}) \rightarrow S_1(\mathcal{M})$ has a continuous section, which sends $p$ to the unique nonforking extension.

**Question 6.** Is there a computable section?

**Question 7.** How complicated is the map $\varphi \mapsto d_p\varphi$ (possibly with uniformity in $p$)?

**Question 8.** Does the existence of a computable section give other computable information for other characterizations of stability?

1.4. **$\kappa^+$-computable categoricity (Knight).**

**Definition.** Let $\kappa$ be a cardinal. Then a set is $\kappa^+$-recursively enumerable when it is $\Sigma_1$ on $L_{\kappa^+}$.

**Definition.** $\mathcal{K}$ is relatively $\kappa^+$-categorical when for any two $\kappa^+$-computable $\mathcal{N}, \mathcal{M}$ of cardinality $\kappa^+$, they are isomorphic in $L_{\kappa^+}(\mathcal{M}, \mathcal{N})$. 
Recall that when an AEC is quasiminimal excellent, it is $\kappa$-categorical for all uncountable $\kappa$.

**Question 9.** Let $\mathcal{K}$ be a quasiminimal excellent class, and $\lambda < \kappa$. Suppose that $\mathcal{K}$ is $\kappa^+$-computably categorical. Must $\mathcal{K}$ be $\lambda^+$-computably categorical?

**Question 10.** Suppose that $\mathcal{K}$ is relatively $\kappa^+$-computably categorical. Must $\mathcal{K}$ be relatively $\lambda^+$-computably categorical?

1.5. $\Sigma$-definable isomorphisms for copies of $\mathbb{C}$ (Goncharov).

**Question 11.** Let $\mathbb{A} = HF(\mathbb{C})$, and suppose $K \cong \mathbb{C}$ is $\Sigma$-definable in $\mathbb{A}$. Is there a $\Sigma$-definable isomorphism?

The answer is yes if we replace $\mathbb{C}$ by $\mathbb{R}$ and both $(K, \oplus, \odot) \cong \mathbb{R}$ and $K \subseteq \mathbb{R}$ hold. It is open if merely $K \subseteq \mathbb{R}^2$.

1.6. $\lambda$-many models of each cardinality $\lambda \geq \aleph_1$ (Greenberg).

**Question 12.** (Assume $V = L$ if it makes things easier.) Suppose that for all $\lambda \geq \aleph_1$ a theory $T$ has at most $\lambda$-many models of cardinality $\lambda$. Must each such model have a $\lambda$-computable presentation?

Such a $T$ is necessarily $\omega$-stable and non-multidimensional. The question is true if $T$ is $\aleph_1$-categorical.

**Question 13.** What if $T$ has finitely many models in $\aleph_1$? (Maybe look at models in $\aleph_n$.)

1.7. Non-abelian free groups (Knight). Consider $n$-generated groups with a single relator of length at most $t$. (For each $t, n$ there are finitely many such groups.) For every sentence $\varphi$ define

$$h_{n,t,\varphi} = \frac{|\{G \in H_{n,t} : G \models \varphi\}|}{|H_{n,t}|}.$$

**Conjecture 14.** The limit $\lim_{t \to \infty} h_{n,t,\varphi}$ exists, and always takes the value 0 or 1, moreover in a way that may depend on $\varphi$ but not on $n$.

**Conjecture 15.** Furthermore, the asymptotically almost sure (a.a.s.) theory determined by this $0 - 1$ law is that of $\mathbb{F}_2$. 
1.8. Standard systems of RCF (Marker).

**Question 16.** What are the possible standard systems of recursively saturated real closed fields? (This is essentially asking: What are the possible sets of cuts of \( \mathbb{Q} \) that are realized in some models?)

The ideal answer might be “all Scott sets”.

1.9. Models of \( \aleph_1 \)-categorical theories. (Andrews).

**Conjecture 17.** For any \( \aleph_1 \)-categorical \( T \) there is an \( n \) such that if \( T \) has a computable model then every countable model has a presentation computable in \( 0^{(n)} \).

Note that if \( T \) is strongly minimal then \( n = 4 \) works.

1.10. Computable prime models (Andrews).

**Question 18** (Millar). Let \( T \) be a decidable theory having countably many countable models. When must the prime model have a decidable presentation? (Note that \( \omega \)-stability suffices.)

**Question 19.** Let \( T \) be a decidable theory having countably many countable models. What do we need to know to build a computable prime model of \( T \)? (Of course \( \omega \)-stability again suffices.)

1.11. Turing degrees of DCFs (Calvert).

**Question 20** (Harizanov). Let \( d \) be a Turing degree. Is there a differentially closed field with a copy that is computable in \( d \) and such that every copy computes \( d \)?


**Definition.** \( \text{Spec}(T) = \{ d : \text{there is a model of } T \text{ computable in } d \} \).

**Conjecture 21.** If \( T \) is totally categorical then \( \text{Spec}(T) \) is a cone.

This is true in a finite language.
1.13. **Model theoretic consequences of Erdős-Rado** (Greenberg).

**Task 22** (Hirschfeldt). *Find proofs in second-order arithmetic of model-theoretic consequences of Erdős-Rado (e.g., forking = dividing in simple theories)*.


**Question 23.** *Let $T'$ be an expansion of $T$ by one constant. Suppose $\cong_{T'}$ is Borel complete. Is $\cong_T$ Borel complete?*

**Question 24.** *Suppose $\cong_T$ is Borel complete. Is $\cong_{T'}$ Borel complete?*