AIM WORKSHOP ON COMPUTABLE STABILITY THEORY PROBLEM SESSION

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1. Questions

1.1. Computable approximability (Calvert).

Definition. $\mathcal{M}$ *is* **computably approximable** *if for every computable $\mathcal{L}_{\omega_1,\omega}$ sentence φ true of $\mathcal{M}$ there is a computable $\mathcal{N} \models \varphi$ such that* $\mathrm{SR}(\mathcal{N}) < \omega_1^{\mathrm{CK}}$, *where* SR *denotes Scott rank.*

Question 1. *Is is the case that every computable structure is computable approximable?*

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Question 2. *Let $\varphi \in \mathcal{L}_{\omega_1,\omega}$ be a satisfiable sentence of quantifier rank α, and suppose that either $\omega_1 > \beta > \alpha$ or $\beta > \alpha + \omega$. Is there an $\mathcal{M} \models \varphi$ such that* $\mathrm{SR}(\mathcal{M}) < \beta$*?*

1.2. **Strongly minimal nontrivial locally modular nonorthogonal groups (Medvedev).**

Let $\mathcal{M}$ be a strongly minimal nontrivial locally modular structure. There is a nonorthogonal interpretable strongly minimal group G.

Question 3. *How difficult is it to find a presentation of G in terms of $\mathcal{M}$?*

Question 4. *How difficult is it to find a presentation of $\mathcal{M}$ in terms of G?*

Consider a three-to-one map $(\mathbb{Q}, +) \leftarrow (\mathcal{M}, \oplus)$ where $(\mathcal{M}, \oplus)$ is strongly minimal.

Question 5. *Must $\oplus$ be definable in some $\mathbb{Q} \times F$, where F is a finite set?*

1.3. **Continuous sections (Miller).**

Consider T stable. Then for all $\mathcal{M} \prec \mathcal{N} \models T$, the map $S_1(\mathcal{N}) \to S_1(\mathcal{M})$ has a continuous section, which sends p to the unique nonforking extension.

Question 6. *Is there a computable section?*

Question 7. *How complicated is the map $\varphi \mapsto d_p\varphi$ (possibly with uniformity in p)?*

Question 8. *Does the existence of a computable section give other computable information for other characterizations of stability?*

1.4. κ^+**-computable categoricity (Knight).**

Definition. *Let κ be a cardinal. Then a set is* κ^+**-recursively enumerable** *when it is Σ_1 on L_{κ^+}.*

Definition. $\mathcal{K}$ *is* **relatively** κ^+**-categorical** *when for any two κ^+-computable $\mathcal{N}, \mathcal{M}$ of cardinality κ^+, they are isomorphic in $L_{\kappa^+}(\mathcal{M}, \mathcal{N})$.*

Recall that when an AEC is quasiminimal excellent, it is κ-categorical for all uncountable κ.

Question 9. *Let $\mathcal{K}$ be a quasiminimal excellent class, and $\lambda < \kappa$. Suppose that $\mathcal{K}$ is κ^+-computably categorical. Must $\mathcal{K}$ be λ^+-computably categorical?*

Question 10. *Suppose that $\mathcal{K}$ is relatively κ^+-computably categorical. Must $\mathcal{K}$ be relatively λ^+-computably categorical?*

1.5. Σ-definable isomorphisms for copies of $\mathbb{C}$ (Goncharov).

Question 11. *Let $\mathbb{A} = \mathrm{HF}(\mathbb{C})$, and suppose $K \cong \mathbb{C}$ is Σ-definable in $\mathbb{A}$. Is there a Σ-definable isomorphism?*

The answer is yes if we replace $\mathbb{C}$ by $\mathbb{R}$ and both $(K, \oplus, \odot) \cong \mathbb{R}$ and $K \subseteq \mathbb{R}$ hold. It is open if merely $K \subseteq \mathbb{R}^2$.

1.6. λ-many models of each cardinality $\lambda \geq \aleph_1$ (Greenberg).

Question 12. *(Assume $V = L$ if it makes things easier.) Suppose that for all $\lambda \geq \aleph_1$ a theory T has at most λ-many models of cardinality λ. Must each such model have a λ-computable presentation?*

Such a T is necessarily ω-stable and non-multidimensional.
The question is true if T is $\aleph_1$-categorical.

Question 13. *What if T has finitely many models in $\aleph_1$? (Maybe look at models in $\aleph_n$.)*

1.7. Non-abelian free groups (Knight).

Consider n-generated groups with a single relator of length at most t. (For each t, n there are finitely many such groups.) For every sentence φ define

$$h_{n,t,\varphi} = \frac{|\{G \in H_{n,t} \,:\, G \models \varphi\}|}{|H_{n,t}|}.$$

Conjecture 14. *The limit $\lim_{t\to\infty} h_{n,t,\varphi}$ exists, and always takes the value 0 or 1, moreover in a way that may depend on φ but not on n.*

Conjecture 15. *Furthermore, the asymptotically almost sure (a.a.s.) theory determined by this $0-1$ law is that of $\mathbb{F}_2$.*

1.8. Standard systems of RCF (Marker).

Question 16. *What are the possible standard systems of recursively saturated real closed fields? (This is essentially asking: What are the possible sets of cuts of $\mathbb{Q}$ that are realized in some models?)*

The ideal answer might be "all Scott sets".

1.9. Models of $\aleph_1$-categorical theories. (Andrews).

Conjecture 17. *For any $\aleph_1$-categorical T there is an n such that if T has a computable model then every countable model has a presentation computable in $0^{(n)}$.*

Note that if T is strongly minimal then $n = 4$ works.

1.10. Computable prime models (Andrews).

Question 18 (Millar)**.** *Let T be a decidable theory having countably many countable models. When must the prime model have a decidable presentation? (Note that ω-stability suffices.)*

Question 19. *Let T be a decidable theory having countably many countable models. What do we need to know to build a computable prime model of T? (Of course ω-stability again suffices.)*

1.11. Turing degrees of DCFs (Calvert).

Question 20 (Harizanov)**.** *Let $\mathbf{d}$ be a Turing degree. Is there a differentially closed field with a copy that is computable in $\mathbf{d}$ and such that every copy computes $\mathbf{d}$?*

1.12. Spectrum of totally categorical theories (Andrews).

Definition. $\mathrm{Spec}(T) = \{d : \text{there is a model of } T \text{ computable in } d\}$.

Conjecture 21. *If T is totally categorical then $\mathrm{Spec}(T)$ is a cone.*

This is true in a finite language.

1.13. Model theoretic consequences of Erdős-Rado (Greenberg).

Task 22 (Hirschfeldt)**.** *Find proofs in second-order arithmetic of model-theoretic consequences of Erdős-Rado (e.g., forking = dividing in simple theories).*

1.14. Borel complexity of isomorphism (Marker).

Question 23. *Let T' be an expansion of T by one constant. Suppose $\cong_{T'}$ is Borel complete. Is $\cong_T$ Borel complete?*

Question 24. *Suppose $\cong_T$ is Borel complete. Is $\cong_{T'}$ Borel complete?*