Invariants in convex geometry and Banach space theory

organized by
Grigoris Paouris, Carsten Schuett, and Elisabeth M. Werner

Workshop Summary

Invariants and their associated inequalities play a central role in convex geometry and geometric analysis on the one side, and in Banach space theory on the other. Moreover, in recent years work is being done on various problems that are related to invariants from other fields. While there have been many interactions between the different communities in the last decade, no systematic effort has been devoted to specifically address interactions between the various invariants occurring in the two fields.

The purpose of the workshop "Invariants in convex geometry and Banach space theory, held at AIM August 20-24, 2012, was to do just that: concentrate on investigating invariants related to a few important problems at the intersection of the two areas. This was achieved through a mix of "tutorial" talks from both areas in the morning that addressed topics relevant to both communities. These talks were given in such a way that they were accessible for all participants and provided the audience with a survey of the main topics. They also led into problem solving sessions by addressing unsolved issues.

Nicole Tomczak Jaegerman lectured on a new proof of a concentration inequality by Grigoris Paouris and raised some questions on the distribution of volume on convex bodies. Peter Pivovarov talked about Gaussian bodies and limit theorems. He also explained how probabilistic estimates for these bodies are related to classical quantities as the affine quermassintegrals and raised several questions related to these quantities. Tutorial lectures given by recent PhDs Joscha Prochnov and Ohad Giladi. They talked about a recent proof by Briët, Naor, Regev on the cotype of the triple tensor product of l_p^n-spaces and explained, as illustrated in the paper by Briët, Naor, Regev, the connections to computer science. There are many open questions in this particular area and some among them were the subject of one of the afternoon discussion groups. Joscha's and Ohad's lecture was followed by a short expose by Steven Taschuk on the K-convexity constant of l_1^n. Monika Ludwig gave an overview of the theory of valuations for convex bodies. In particular, she raised issues of expanding the theory beyond its classical realm, by considering other than real valued valuations, so for instance vector valued ones. Artem Zvavitch gave a talk on intersection bodies and several open questions on this subject. In particular he focused on the uniqueness of the fixed point of the operator I and the role of convexity in these problems. Stanislaw Szarek lectured about quantum information theory: Many of the sets appearing naturally in quantum information theory are convex and also live in very high dimensions. Thus methods from convex geometric analysis, which deals exactly with convexity phenomena in high dimensions, promise to be applicable to questions posed in quantum information theory. Indeed, such approaches have been successful recently and Szarek illustrated some of the successes and raised more problems where such an approach might work as well. Alexander Litvak lectured on some of the recent developments in the asymptotic theory of random matrices generated by log-concave densities and their applications as the recently obtained Chevet inequality for 1-unconditional log-concave matrices. Herman König gave a talk on the sharp constants on Kinchine inequalities and posed some related questions. Mathieu Meyer lectured on some recent developments on Mahler conjecture. He gave the proof of a theorem by Reisner, Schütt and Werner which says that if a convex body has just one point on its boundary with strictly positive Gaussian curvature, then the volume product of the body and its polar cannot be minimal. This is the strongest result to date indicating that the volume product is minimal for polytopes, a conjecture that is still open. Mathieu also lectured on the functional analogue of the Reisner-Schütt-Werner result, something he proved with Y. Gordon. We want to point out that both, the Reisner-Schütt-Werner result as well as the Meyer-Gordon result was started at a previous AIM workshop.

The afternoons of the workshop were almost entirely devoted to discussions, mostly in smaller groups. On Monday afternoon, in a long and very active session, open problems and questions were solicited from the participants. On the afternoons of Tuesday - Friday smaller groups of 3 - 7 participants worked on and discussed the open problems gathered on Monday. The list of problems which were raised will appear on AIM's webpage. We now describe some of the topics and problems discussed in the afternoon sessions in more detail.

K-convexity constant for non-symmetric sets and bounds for MM^*

This group discussed some approaches to define the K convexity constant for non-symmetric sets and the known techniques for upper-bounding the MM^* product. One of the conclusions was that in order to progress on these questions, a better understanding of the symmetric case is required. With that in mind, the group discussed the question of whether the K-convexity constant and the MM^* product are indeed equivalent, even for symmetric bodies (usually the former is used as an upper bound for the latter). Some new positive evidence was noted. As a follow up to a lecture presented earlier during the workshop, a unified argument giving (precise) lower bounds for the two invariants for the simplex and the unit ball of ℓ_1 was produced. The argument gives hope for a more conceptually transparent variant of Bourgain's example of a space (or, equivalently, symmetric set) with the worst possible constant. Some new insights in that direction were acquired.

Type and cotype for tensor products

It is an open question for more than 30 years whether the projective tensor product $\ell_p \otimes \ell_r$ has cotype 2 for $1 \leq p, r \leq 2$? (The case $p = r = 2$ was solved by N. Tomczak-Jaegermann.) It turned out that the same questions for triple tensor products is relevant to coding theory. The invariant *volume ratio* is related to cotype 2 and we estimated it for triple tensor products during the group sessions.

Distribution of volume in $G_{n,k}$

This group has discussed some affine isoperimetric inequalities that are open such as Lutwak's conjecture on affine quermassintegrals and extensions of Lutwak-Yang-Zhang's p-centroid inequality to negative values of p.

Intersection bodies and invariants

This group discussed questions related to intersection bodies as well as volumetric invariants that characterize convex bodies. In particular, effort has been made to construct an example of a convex body such that the Banach-Mazur distance to the Euclidean ball increases when one applies the Intersection operator.

Projection bodies and invariants

The group studied the problem of giving a lower bound to the volume of the polar of the projection body of a convex body. This problem has already a solution for general convex bodies, but giving a sharp lower bound in the particular case of centrally symmetric bodies is still an open problem. In particular, the group studied the case of zonoids, and although they did not yet solve the problem, big progress was made concerning this approach.