

# CYCLIC HOMOLOGY AND SYMPLECTIC TOPOLOGY

organized by

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## Workshop Summary

This workshop was prompted by the recent preprint [2] of Bourgeois, Ekholm, and Eliashberg giving a surgery formula for contact homology and related invariants of Stein manifolds. It brought together symplectic topologist along with experts in topological field theory in order to understand (i) the connection with conjectures described in [3] by Seidel which relate the symplectic homology groups of the total space of a Lefschetz fibration to the Hochschild homology of a certain category built from an explicit collection of Lagrangian contained in a smooth fibre, as well as (ii) explore consequences for the algebraic structures on contact and symplectic homology.

### Workshop Activities

The mornings of the first two days consisted of talks by Bourgeois and Ekholm outlining their preprint [2], and by Ganatra and Maydanskiy giving a dictionary relating these results to the conjectures of Seidel. In the afternoons, two groups formed, one focused on understanding the symplectic and contact homology of manifolds obtained by Legendrian surgery on  $S^3$  or connected sums of  $S^1 \times S^2$ , while the other used the setting of cotangent bundles to try to understand a picture of these invariants from the point of view of topological field theory, with symplectic homology expressed as Hochschild homology.

On Wednesday morning, Oancea presented his work [1] on expressing contact homology as an  $S^1$ -equivariant version of symplectic homology, while Seidel further explained the connection with Fukaya categories of Lefschetz fibrations. In the afternoon, the topological field theory group compared the algebraic operations on cyclic homology with those expected on contact homology, another group sought to find invariants to distinguish symplectic manifolds with vanishing symplectic homology, while a third considered a generalisation of Seidel's conjectures to the case of Morse-Bott singularities.

On Thursday, Perutz explained his proof of a conjecture of Seidel computing the Floer homology of the monodromy symplectomorphism acting on the fibres of a Lefschetz fibration from knowledge of the Lagrangian Floer homology of the vanishing cycles. McLean showed how the Floer homology of the monodromy can be used to compute the symplectic homology of the total space. In the afternoon, one group continued the study of operations on contact and  $S^1$ -equivariant symplectic homology, while another explored some ideas proposed by Eliashberg for using the work of [2] to construct symplectic manifolds with various properties.

The final day started with a talk by Abouzaid on using symplectic homology to understand the Fukaya category of Liouville domains, followed by Teleman giving a general picture of topological field theories and their deformation theory. In the afternoon, one group discussed the Grothendieck group of Fukaya categories, while another discussed strategies for producing exotic symplectic structures in higher dimensions by applying the surgery formulae to Legendrian surgery on jet spaces.

In the organizers' opinion, the workshop was quite successful, with fruitful dialogue initiated between researchers in various areas (notably symplectic topology and homological algebra) on the workshop topic, and progress made in several areas of emphasis (see below).

### Achievements

- In the case a single handle is attached to a subcritical manifold, we extended the results of Bourgeois-Ekholm-Eliashberg to give an equivalence of  $A_\infty$  structures between the Legendrian contact homology algebra of the Legendrian spheres before handle attachment and the wrapped Floer homology of the co-core Lagrangian disc after surgery.
- We clarified the status of the string cobracket of Chas and Sullivan as an operation which does not follow the topological field theory formalism. In particular, Seidel described a construction of such a cobracket on  $S^1$ -equivariant symplectic homology, as a secondary operation coming from the vanishing of the coproduct on the “positive” part of symplectic homology.
- Ekholm and Ng produced examples of simple Legendrian knots in  $S^1 \times S^2$  which can be used to construct exotic smooth structures on  $R^8$  and perhaps  $R^6$ .

### Outlook

- Is there an extension of the Topological Field Theory formalism which produces operations analogous to the cobracket on contact homology?
- Can the approach of [2] determine the product structure on symplectic cohomology? In particular, it should agree with the product on the Hochschild cohomology of the Legendrian contact homology algebra? What about other algebraic structure on symplectic and contact homology?
- There is now a large number of constructions producing symplectic manifolds with vanishing symplectic homology. Are there any invariants distinguishing them?

### Bibliography

- [1] F. Bourgeois and A. Oancea, *An exact sequence for contact- and symplectic homology*, available at [arXiv:0704.2169 \[math.SG\]](#) .
- [2] F. Bourgeois, T. Ekholm, and Y. Eliashberg, *Effect of Legendrian Surgery*, available at [arXiv:0911.0026 \[math.SG\]](#) .
- [3] P. Seidel, *Symplectic homology as Hochschild homology*, available at [arXiv:math/0609037](#) .