

DEFINABILITY AND DECIDABILITY PROBLEMS IN NUMBER THEORY

organized by

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Workshop Summary

The main goal of the workshop was to consider new and old problems of definability and decidability in Number Theory to see if any progress can be made by the group. To that effect and in keeping with the AIM style of workshops, all afternoons were dedicated to work in groups on problems.

The following talks were presented during the morning sessions. Monday morning Russell Miller spoke on a Computability approach to Hilbert's Tenth Problem for \mathbb{Q} , and Hector Pasten spoke on Büchi's problem and its connection to uniform definability across positive characteristics. Tuesday morning Jochen Koenigsmann discussed a model theoretic approach to the definability problems, in particular to showing that \mathbb{Z} does not have a Diophantine/ existential definition over \mathbb{Q} . Moshe Jarden discussed decidability in infinite extensions. Wednesday morning Angus Macintyre spoke on definability in weak arithmetic, and Jennifer Park spoke on a universal definition of the rings of integers over number fields. Thursday Jeroen Demeyer spoke on r.e. sets vs Diophantine sets over polynomial rings in positive characteristic. Friday morning Kirsten Eisentraeger spoke on definability and decidability over function fields of positive characteristic.

Monday afternoon Tom Scanlon led a problem session which provided the basis for the working groups for the rest of the week. A complete list of suggested problems follows the report. Tuesday afternoon the working groups were formed. The groups continued their work in the afternoon during the rest of the workshop. A more detailed description of the group projects and other preliminary results can be found below.

- HTP for subrings of \mathbb{Q} : Kirsten Eisentraeger, Russell Miller, Jennifer Park, Tom Scanlon, Alexandra Shlapentokh. The group obtained a preliminary proof that there exist c.e. subrings of \mathbb{Q} with the natural density of the inverted prime equal to arbitrary computable real number between 0 and 1 such that HTP of the ring is Turing equivalent to HTP over \mathbb{Q} .
- Mapping graphs into fields: Russell Miller, Thomas Scanlon, Alexandra Shlapentokh, Jennifer Park, Bjorn Poonen. A preliminary proof has been obtained.
- Existential undecidability of $K(t)$ for an arbitrary field K in the language $\{0, 1, +, \cdot, T\}$, where $T(f) \leftrightarrow f \notin K$? The group consisted of Kirsten Eisentraeger, Thanases Pheidas, Hector Pasten, Natalia Garcia Fritz.
- Given $\mathbb{Q}^* \subseteq \mathbb{Q}^{**}$, both elementary equivalent to \mathbb{Q} (in the language of rings). Does it imply that $(\mathbb{Q}^*)^{\text{alg}} \cap \mathbb{Q}^{**} = \mathbb{Q}^*$ (i.e. is \mathbb{Q}^* relatively algebraically closed inside \mathbb{Q}^{**})? Jochen Koenigsmann, Angus Macintyre, Paola d'Aquino, Deirdre Haskell, Will Johnson.

- r.e. sets are Diophantine, uniformly across rings of polynomials of positive characteristic. This group consisted of Jeroen Demeyer, Joseph Flenner, Alexandra Shlapentokh, Xavier Vidaux. The group obtained a preliminary proof.
- For K and L finitely generated fields, does $K \equiv L$ imply $K \cong L$? Does there exist a formula $\varphi(x, y)$ such that for any finitely generated field K and every rank one divisorial valuation v on K , there exists a parameter a such that $\mathcal{O}_v = \text{set } b \in K \varphi(b, a)$? This group consisted of Joseph Flenner, Tom Scanlon, and Adam Topaz.
- Some problems related to infinite extensions: Moshe Jarden, Aharon Razon, Carlos Videla. A paper is in preparation.
- Jochen Koenigsmann has answered a question of Abraham Robinson which was brought to his attention by Carlos Videla. A pre-print of the paper is available at arXiv:1309.7138.