Algebraic and combinatorial structures in exactly solvable models

organized by
Chiara Franceschini, Jeffrey Kuan, and Zhengye Zhou

Workshop Summary

Overview

This workshop, held from June 15 to June 19, 2026 in Pasadena, was devoted to drawing connections in between integrable probability, quantum groups and Hecke algebra, and combinatorics. It was sponsored by the American Institute of Mathematics (AIM) and the National Science Foundation (NSF). We had 23 participants: Gioia Carinci (Università di Modena e Reggio Emilia), Colin Defant (Harvard University), Evgeni Dimitrov (University of Southern California), Chiara Franceschini (Università di Modena e Reggio Emilia), Promit Ghosal (University of Chicago), Vadim Gorin (University of California, Berkeley), Pallav Goyal (University of California, Riverside), Wolter Groenevelt (Delft University of Technology), Jimmy He (Ohio State University), David Keating (University of Illinois Urbana–Champaign), Jeffrey Kuan (Ohio State University), Eunghyun Lee (Nazarbayev University), Olya Mandelshtam (University of Waterloo), Chihiro Matsui (The University of Tokyo), Sevak Mkrtchyan (University of Rochester), Nimisha Pahuja (ICTS Bengaluru), Marianna Russkikh (University of Notre Dame), Anastasiia Trofimova (Gran Sasso Science Institute), Bálint Vető (Budapest University of Technology and Economics), Kevin Yang (Harvard University), Oleg Zaboronski (University of Warwick and IHES), Xincheng Zhang (Caltech), Zhengye Zhou (University of Southern California).

Morning Lectures

We had 6 Introductory Lectures from Jeffrey Kuan and Oleg Zaboronski on Monday, Wolter Groenevelt and Olya Mandelshtam on Tuesday and Balint Veto and Chihiro Matsui on Wednesday. In addition, we have more technical seminars from Kevin Yang and Jimmy He on Thursday and Vadim Gorin and Marianna Russkikh on Friday.

On the first day, Jeffrey Kuan gave an introductory talk about quantum groups and their connection with the Yang–Baxter equation. As a prototype example, he considered the asymmetric exclusion process with jump rates α to the left and β to the right. Moreover, he introduced the Lie algebra $\mathfrak{sl}_2$ and its deformation via the Drinfeld-Jimbo quantum group. The second talk of the morning was given by Oleg Zaboronski, who discussed Markov duality and its relation to interacting particle systems. After recalling the definition of duality, he showed several examples known in the literature, such as coalescing and annihilating random walks on $\mathbb{Z}$ and zero-range processes. Finally, he presented a new result concerning a duality with Pfaffian point processes. On Tuesday, Wolter Groenevelt gave the first talk on the algebraic approach to duality. He showed how to relate the coproduct of the Casimir element of the Lie algebra $\mathfrak{sl}_2$ to the Markov generator of an interacting particle system. From this observation, he then explained how to construct orthogonal duality relations under the assumption that the particle system is reversible. The second talk was given by

Olya Mandelshtam, who discussed symmetric and non-symmetric functions, in particular how the partition function of the asymmetric exclusion process can be obtained as a specialization of Macdonald polynomials. On Wednesday, Balint Vető gave the first talk about the Kardar-Parisi-Zhang equation and the notion of a universality class. He explained how to recover the KPZ equation for the height function of the asymmetric exclusion process in the hydrodynamic limit under weakly asymmetric scaling. He also showed how to interpret this ill-posed equation via the Cole–Hopf transformation and explained the so-called KPZ universality class conjecture. The second talk of the day was given by Chihiro Matsui who discussed integrability for quantum systems and stochastic processes. She showed how to connect the asymmetric simple exclusion process with the XXZ Heisenberg spin chain via a gauge transformation. She discussed Yang-Baxter solvability and ended by introducing the notion of partial solvability. On Thursday, Kevin Young discussed the role of loop equations in random matrix theory. He began with the model introduced by Wigner in the 1950s, whose aim is to study the macroscopic behavior of the empirical measure of the eigenvalues of a random matrix, as well as the law of eigenvalue gaps in the bulk and in the edge. He then explained how the Stieltjes transform can be analyzed through a first-order loop equation, and concluded by presenting a new flow-based approach to the problem, based on the study of time-dependent quantities. The second talk was given by Jimmy He. He presented a talk on the six-vertex model and its integrable structure. He discussed the role of the Yang–Baxter equation in establishing exact solvability and explored the connections between the six-vertex model and families of special symmetric polynomials arising in algebraic combinatorics and representation theory.
On Friday, Vadim Gorin presented recent results on Gaussian free field (GFF) fluctuations arising from random lozenge tilings. The talk focused on the asymptotic behavior of random tiling models, illustrating how the Gaussian free field emerges as a universal scaling limit for height function fluctuations. Marianna Russkikh presented recent developments on the dimer model and t-embeddings. The talk introduced t-embeddings as a geometric framework for the analysis of planar dimer models and discussed their role in understanding scaling limits and height fluctuations, highlighting connections between discrete complex analysis, integrable probability and random surface models.

List of open problems

During the first afternoon we have listed the following open problems which were brifley introduced and discussed thanks to the presence of a moderator, David Keating.

(1) Three-particle reaction–diffusion model
(2) Transition probabilities for the open ASEP
(3) Connection between the open ASEP and the Sachdev-Ye-Kitaev (SYK) model
(4) Multi-species long-range swap model
(5) Multi-lane model: stationary measures and scaling limits of its height function
(6) Find a Markov chain on multiline queues that projects to the multi-species ASEP on a ring
(7) Bounded Cauchy and near-rectangular identities for Macdonald polynomials
(8) Momentum redistribution model reversible with respect to the hyperbolic secant distribution: underlying algebra and duality relations
(9) Find spin-chain Markov processes associated with the strange-series representations of $\mathcal{U}_q(\mathfrak{su}(1,1))$

(10) Find a Markov diffusion on $SU(2)$
(11) Probabilistic proofs for families of integrable PDEs: connections with vertex models and the Yang–Baxter equation
(12) Connection between Schur-Weyl duality and Markov duality
(13) Limit shapes for coupled tilings
(14) Algebraic description of the SIR duality with a two-layer random walk
(15) Loop equations for Lie groups other than type A
(16) Stationary measures of multi-species zero-range processes (ZRPs)
(17) Understand duality for matrix-valued orthogonal polynomials arising from interacting particle systems
(18) Bernoulli/exponential first-passage percolation (FPP) and its relation to continuum models such as the Brownian web
(19) Integrable versions of massive coalescence models (Kolmogorov's model), for which the 5/3 conjecture fails
(20) Random band matrices: understanding localization behavior at transition points

Developments on the open problems which have been discussed by the participants

3. This project aims to consolidate the proposed correspondence from open ASEP to open KPZ and JT gravity/Schwarzian theory into a rigorous research program. The framework is organized around seven key ingredients that describe a common continuum object from three complementary perspectives: the algebraic viewpoint of the exclusion process, the probabilistic viewpoint of the two-layer Brownian Gibbs ensemble, and the geometric viewpoint of Schwarzian boundary theory. During the workshop, the central ingredient was formulated as a precise main theorem supported by a sequence of lemmas and propositions, and a key conceptual point concerning the distinction between the interval and thermal circle measures was clarified, leading to a sharper formulation of the central conjecture and its remaining technical challenges. Connections with several recent developments in the field were also identified, opening opportunities for future multi-institutional collaborations.

4. This model has been recently introduced as a generalization of the totally asymmetric exclusion process. Not much is known about it besides the fact that it is integrable, i.e. it satisfies the Yang-Baxter equation when the jump rates are site homogeneous. The group discussed some interesting questions, such as the integrability for a generalized version with inhomogeneous jump rates, the invariant measure on a ring and asymptotics for the position of the second class particle and for the partition function of the nth right most first class particle with initial step condition.

5. The multilane model is a good candidate for analyzing coupled 1D KPZ-type equations arising from linearizing conservation laws around a stationary state. We considered a two-lane asymmetric simple exclusion process with transition rates depending on the particle number in both lines, but found that its stationary state admits no factorized representation - a significant obstruction to analytic treatment. We therefore turned to the simpler setting of Ferrari–Sasamoto–Spohn (2013), who instead considered a single-lane, three-species exclusion process (the AHR model)

whose stationary state is exactly given by a matrix-product ansatz. This exact solvability allowed them to analytically compute the mode-coupling constants and confirm via simulation that both KPZ peaks scale according to the exact KPZ scaling function.

7. The group discussed the TAZRP-polynomials and nonsymmetric Macdonald polynomials by comparing bosonic and fermonic vertex models, which on certain domains had partition functions equal to these polynomials. It has been discussed what sort of results they thought may be provable about the polynomials using the vertex models.

9. The goal of this problem was to construct a Markov process from the strange series representations of $\mathcal{U}_q(\mathfrak{su}(1,1))$. The group succeeded in finding an interacting particle system where particles jump to nearest-neighbor sites with asymmetric rates. The group also proved reversibility of the model by imposing the detailed balance condition. The group decided to keep working on this project and to investigate whether it is possible to find a Markov duality relation, compute the currents of the model and understand what happens when equivalent algebraic representations are considered.

15. This group was working on developing discrete loop equations (or Nekrasov equations, or discrete Dyson-Schwinger equations) for the particle systems with a wall. This can be treated as finding BCD-type analogues for the type A equations. The key challenges are in finding the particle interactions for which the equations have a chance to exist, and in figuring out proper forms of holomorphic observables for these interactions. During the workshop, the group succeeded in three individual cases, corresponding to types B,C, and D, where interactions arise from Weyl formulas for dimensions of irreducible representations of symplectic and orthogonal groups. Future tasks involve unifying the three formulas into one and asymptotic analysis.

16. The group discussed multi-lane interacting particle processes in which jump rate for a particle in some lane depends on the local configuration of particles in all the other lanes. They looked at physics papers which defined specific cases of this type of particle process and made claims about their properties. The initial focus was on identifying the mathematical techniques needed to establish these claims rigorously.