

ANALYTIC THEORY OF $GL(3)$ AUTOMORPHIC FORMS AND APPLICATIONS

organized by

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Workshop Summary

INTRODUCTION. The original motivation for organizing this workshop came from the vision of many experienced researchers that there is great potential in harmonic analysis on $GL(n)$ for applications to classical problems and studying new questions in analytic number theory. This was particularly assured by remarkable success in applications of $GL(2)$ in the last three decades. In spite of numerous spontaneous attempts to employ higher rank analysis the results obtained so far are only scratching the surface of this large space. Perhaps the reason for slow development can be explained by lack of closer communication between researchers in these so distinct subjects. Therefore one of the goals of the workshop was to create an opportunity for specialists in analytic number theory to meet those who are more knowledgeable in automorphic forms beyond $GL(2)$. To get started it was a practical idea that we should focus particularly on the spectral theory on $GL(3)$. Of course, understanding things in a more general context also helps, so there was considerable presentation of $GL(n)$ for any n . The activity was kept truly in a workshop style (which should be credited to AIM); there was no program fixed in advance, speakers were asked to give talks on short notice. In this way a lovely conversational style was achieved during the talks which continued over entire days (including lunch time). We were nicely surprised to notice that many younger participants were quite advanced in capturing the ideas which were inevitably presented in a sketchy manner. Not only was the educational component of the workshop so important, but also getting fresh inspiration by sharing different views was in our mind. Certainly one can say that the workshop succeeded in this regard, because presentations and conversations in afternoon groups were very intense and popular.

SHORT DESCRIPTION OF TALKS. Every morning there were talks given by selected participants requested by the organizers to cover material for subsequent activity in the afternoon (we thank the AIM members for guiding us and for hints on how to proceed in such improvised atmosphere). These talks lasted for hours and the audience was encouraged to interrupt with questions and comments. Indeed this was happening more often as time was passing, surely for the benefit of the listeners.

The first talk on Monday given by Philippe Michel was about Hecke operators in a lovely setting of lattices. This approach was particularly instructive to see how the Hecke operators act on automorphic forms on $GL(n)$ without explicit use of adelic language. Quite illuminating was the link to invariant differential operators.

The second talk on Monday given by Steve Miller was about Eisenstein series on $GL(3)$. Since these series are building blocks for spectral resolution of the full L^2 space of automorphic forms, it was important to grasp their features right at the beginning. Fourier expansion was given explicitly only for $SL(3, \mathbb{Z})$, which gives a glimpse of what to expect

in the case of congruence groups. Many questions were raised by the audience concerning the congruence subgroups. These were further addressed in afternoon split sessions. The presentation was enhanced by comments from experts in this field, particularly by Dorian Goldfeld and Daniel Bump.

All talking on Tuesday morning was by Akshay Venkatesh about Weyl Law distribution of eigenvalues on manifolds. Some surprisingly sharp results were derived from the pre-trace formula; the spectral gap was discussed as well as intricate issues of multiplicity.

Dinakar Ramakrishnan and Frydoon Shahidi talked on Wednesday. They spoke adelicly most of the time, yet they campaigned to convince novices that this was the right way to do things in higher rank situations. Shahidi concluded his talk with a review of the results which were accomplished in $GL(2)$ (demanded in analytic number theory) by indirect appeal to L-functions (such as the symmetric powers), whose natural habitat contains special groups. In particular he praised highly the progress in the Ramanujan-Selberg conjecture at finite and infinite places. This was the most powerful demonstration of high rank harmonic analysis in action.

On Thursday we heard Paul Garrett showing and gracefully speculating on various constructions, again adelicly, including the Eisenstein series and triple products L-functions. It was a desire of many participants to have a clear translation between local (adelic) objects and global constructions which are more familiar among analytic number theorists. While a lot of progress was made by exchanging views between different groups, there is still need for more effective instructions.

On Friday there were three talks. The first two were given by Roman Holowinsky and Kannan Soundararajan. They presented (in complementary fashion) some aspects of their joint achievement, the unconditional proof of the Quantum Unique Ergodicity Conjecture. This is a spectacular result which borrows many ingredients from analytic number theory and automorphic forms. No doubt they have stolen the show. The third talk was by Xiaoqing Li about her breaking convexity bounds for certain L-function of degree three. Her result is also relatively new. The subconvexity bounds for L-functions are critical for applications, and a whole workshop at AIM was devoted to these topics in October 2006.

AFTERNOON WORKING GROUPS. Three times after morning talks a selection of interesting topics was proposed by the participants for closer considerations by small groups in the afternoon sessions. Next day younger listeners in each group (rather than the leading presenter) gave brief description to all participants about what was discussed in their group. For example we list here a few (but not all) subjects which attracted many people;

- Quantitative Analysis of Weyls Law
- Kloosterman sums in $GL(3)$
- Special Functions and their Integral Transforms
- Constant Terms in Eisenstein Series and The non-vanishing of L-functions
- Computational Problems in $GL(3)$
- Voronoi-type Summation Formula in $GL(3)$ and its Applications
- Kuznetsov-type Spectral Formula in $GL(3)$
- Triple Shifted Convolution
- Relative Trace Formula (Geometric Side, Moments)
- Root Number of L-functions.

During these sessions some lively interactions have taken place and several projects have been set up for the future (bringing together many of the participants in groups of similar interest). So it is expected that some of these activities will carry on long after the workshop.

REFLECTIONS ON OTHER ISSUES. One of the highlights of the workshop was the problem session (all the problems are recorded in a separate report), which was very encouraging, because it manifested that young people have already some feeling and hope for pursuing their research in modern analytic number theory based on harmonic analysis of high rank. Actually some problems were solved on the spot during the workshop. Nobody expected to receive immediate gratification from the activity during the workshop, but rather a new impetus to work on the problems after the meeting. We are looking forward for the next gathering at AIM in a few years to continue this favorite and fruitful trend.