Gromov-Witten invariants and number theory

organized by
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Workshop Summary

1. Workshop Activities

1.1 Lectures

- Don Zagier: *Elliptic modular forms and their applications I* The aim of the first lecture was to give a steep, modern introduction into the properties of modular forms and the operations on thems. Zagier introduced the basic notions of the rings of holomorphic, meromorphic, almost holomorphic and quasi and vector valued modular forms of $PLS(2, Z)$. He discussed the Hecke operators, the Mass and Serre derivative and the Rankin Cohen brackets as well as the differential equations fullfiled by modular forms. His lectures where enhanced with many explicite examples, which were further discussed in the afternoon.
- Yongbin Ruan *Landau Ginzburg models and Mirror Symmetry in families* Ruan explained how mirror symmetry in famillies is related to modularity. He described the expansion of the prepotential around the discriminant locii in compact and noncompact families of Calabi-Yau manifolds in particular the large radius Gromov-Witten Expansion and the Orbilfold Gromov-Witten expansion at the Landau Ginzburg point.....
- Don Zagier: *Modular forms and their applications II* After finishing the review of Jacobi forms, Zagier devoted his second lecture to mock modular forms. A short historical review of Ramanujans 17 examples of mock theta functions was followed by the definition of mock modular foms and the three main methods to construct them together with their shadows: Using Apell Lerch sums, Quotients of indefinite binary theta series by unary theta series and the Fourier coefficients of meromorphic Jacobi forms. He focussed on the later, because they are directly related to the micro states yielding the entropy of N=4 super symmetric black holes and wall crossing phenomena, which concluded his lecture.
- Jan Stienstra: *Holomorphic anomaly equation and Beauville families* Jan Stienstra introduced the geometry of rational elliptic surfaces, the Kodaira classification of their singular fibres. He explained Beauvilles classification of stable families of elliptic curves over P^1 with exactly four singular fibres and generalizations discussed by Schmickler-Hirzebruch. As was found by Zagier a subset of Apery like equations that allow for solutions with integer coefficients are related to these cases essentially through mirror symmetry. A specialized version of the holomorphic anomaly equation can be used to calculate the genus zero Gromov-Witten invariants for local Calabi-Yau spaces based on these rational elliptic surfaces.

- Albrecht Klemm *The holomorphic anomaly equation* Albrecht Klemm reviewed the B-model approach to the calculation of the topological string B-model amplitudes and their mirror symmetry relation to Gromov-Witten-, Donaldson Thomas- and stable pair invariants of the topological A-model. He showed how the holomorphic anomaly equation implies the quasi modularity of the amplitudes for local Calabi-Yau manifolds and together with the gap conditions fixes these amplitudes. The choice of the integer sympletic basis and the relation of the wave function transformation of the topological partition functions under a symplectic change of basis and the holomorphic anomaly was explained. The lecture ended with the calcucaltion of refined stable pair invariants using refined holomorphic anomaly equation and refined boundary conditions
- Sheldon Katz *Sympletic invariants on Calabi-Yau manifolds and localization* Sheldon Katz provided first the mathematical definition of the topological A-model invariants in terms of integrals over virtual cycles in the moduli space of stable maps, of stable sheafs and in particular stable pairs. He reviewed correponding virtual deformation and obstruction theory calculated the expected virtual dimesnions of the moduli space and showed then how these invariants can be explicetly evaluated using localizations methods for toric Calabi-Yau spaces. He focussed at the end of the talk on the virtual intersection theory on the moduli space of stable pairs and explained how this can be refined using a virtual version of the Bialinicki-Birula decomposition if the target space admitts an $U(1)$ isometry.
- Sameer Murthy *Quantum black holes and Mock modular Forms and Wall Crossing* Sameer Murthy completed the second part of Don Zagiers story about meromorpic Jacobi forms and mock modularity from the physical perspective. In particular he discussed the semi-calssical and the microscopic entropy of $N = 4$ black holes and why the micro states are counted by meromorphic Jacobi forms and how the mock modularity is related to the wall crossing and the
- Jeff Harvey and Miranda Cheng *Umbral Moonshine* Jeff Harvey and Miranda Cheng explained the notation of umbral Moonshine, which connects the representation theory of the large sporadic finite groups to mock modular counting functions.

2. Projects

The participants were selected so that they initiate or finalize projects.

2.1 Emerging projects Don Zagier and Jan Stienstra are working on a project related to the holomorphic anomaly and the Apery like equations that featured in Stienstras talk. Albrecht Klemm and Minxin Huang working on the solutions of the topological string on compact elliptically fibred Calabi-Yau spaces.

2.2 Finished Projects Minxin Huang and Albrecht Klemm settled at AIM problems which were essential to implement the holomorphic anomaly equation and the gap condition calculate refined stable pair invariants for del Pezzo surfaces and the $\frac{1}{2}K3$. This work is now finished with reference to the AIM support. "Refined stable pair invariants for E-, M- and [p,q]-strings" by Minxin Huang, Albrecht Klemm and Maximillian Poretschkin. A second Project was discussed by Jeff Harvey and Miranda Cheng and helped to finalize the paper "Umbral Moonshine and the Niemeier Lattices" with John F. R. Duncan.