Integrable systems in Gromov-Witten and symplectic field theory

organized by
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Workshop Summary

The workshop had the AIM format of two lectures in the morning and discussions and problems solving in the afternoon. Most of the participants came from European and American institutions. They can be divided into 3 groups depending on their research interest: algebraic geometry, symplectic geometry, and integrable systems. Unfortunately some of the experienced researchers, such as Igor Frenkel, Motohico Mulase, Yongbin Ruan, Ezra Getzler, and Alexander Givental could not come for various reasons, often related to their teaching duties. Nevertheless, despite their young age, we had some very strong and active mathematicians, such as S. Shadrin, D. Zvonkine, R. Cavalieri just to name a few. Overall when it comes to making an account of the results relevant for solving the main problem of the workshop and formulating problems that should be addressed in the immediate future, the goals of the workshop were achieved.

The main problem of the workshop was to compute certain invariants in Symplectic Field Theory (SFT), which are known to give rise to a quantum integrable system. The latter is also known to be a quantization of the so called genus-0 or principal hierarchies introduced by B. Dubrovin for the purposes of Gromov–Witten (GW) theory. We started the workshop on Monday with two introductory talks by Paolo Rossi and Boris Dubrovin who explained the main problem to all participants. The next day there was a talk by D. Zvonkine about a formula due to R. Hain expressing certain locus in the moduli space of curves of compact type in terms of boundary divisor classes. Later on Y. Eliashberg explained to us the SFT theory of the cylinder. Apparently the work of R. Hain allows us to compute the genus-1 correction to the SFT invariants. Moreover, there is an indication that Givental's symplectic loop group formalism can be used in SFT as well. With the genus-0 part being known already, this becomes quite an interesting problem for the specialists in integrable systems. Namely, to find an axiomatic theory of quantum integrable systems which in particular can be applied in SFT. One of the problems that we tried to work out is to check that Hain's prediction is correct in the case of the SFT of the cylinder. We could not finish this exercise, so it remained as one of the problems for the immediate future.

We also followed another direction in GW theory. Namely, the construction of integrable hierarchies that govern higher genus GW invariants. This is yet another deformation of the genus-0 hierarchies. We had a talk by S. Shadrin and the introductory talk by B. Dubrovin, who explained to us two different ways of recovering the higher genus theory. In both approaches there is a fundamental notion that serves as a base for the construction. In Shadrin's approach this is the notion of a Cohomological Field Theory as well as the quantization formalism of Givental, while in the approach of Dubrovin (and his collaborator Y. Zhang) the base is the notion of a Frobenius manifold. This is another open question in SFT what kind of structure will govern the higher genus corrections. Also, is it possible to extend the SFT quantization, namely to obtain a quantization not only of the genus-0 part, but of the entire integrable hierarchy. In particular, even in the case of the KdV hierarchy, is it possible to quantize the first bracket.

In the afternoons we formed several small groups, according to the AIM guidance, where we discussed specific problems or clarified further some of the results that were announced in the morning. Some of the topics were: the Ablowitz-Ladik integrable hierarchy and its application to GW theory, Givental's quantization operator, Hain's formula for the principal divisor locus in the moduli space of curves, as well as the related problem of constructing the tautological family of Jacobians on the Deligne-Mumford moduli space, Frobenius structures related to local symplectic field theory of a Reeb orbit, higher structures in rational symplectic field theory, etc.

In conclusion, SFT is a fairly new subject and it still lacks computational techniques. In GW theory one can use algebraic geometry to make explicit computations, but in SFT the applications of algebraic geometry are limited. This is probably one of the main obstacles. We do not have enough examples where a non-trivial computations with an interesting answer can be obtained. Nevertheless, it looks that a certain progress was made and it is quite plausible that the methods of integrable systems will become an essential computational technique in SFT.