High-dimensional phenomena in discrete analysis

organized by
David Conlon, Sarah Peluse, and Yufei Zhao

Workshop Summary

Project group summaries

During the workshop, participants heard talks and discussed a wide range of topics related to additive combinatorics, discrete geometry, and extremal graph/hypergraph theory, with focus on high dimensional phenomenon. Here are some summaries from group discussions during the week.

Roth with constrained differences
(contact: speluse@umich.edu).

Let p be an odd prime and D a proper subset of $\mathbf{F}_p$ containing 0. We studied the problem of obtaining bounds on the size of subsets of $\mathbf{F}_p^n$ lacking nontrivial 3-term arithmetic progressions

$$x, x+y, x+2y; y \neq 0$$

with common difference $y \in D^n$. One particular case of this problem was considered by Bhangale, Khot, and Minzer, who proved that when $D = \{0,1,2\}$, then any subset $A \subset \mathbf{F}_p^n$ free of nontrivial 3-term arithmetic progressions with common difference in D^n satisfies the bound $|A| = O\left(\frac{p^n}{\log\log\log n}\right)$. We worked out an argument that improves this bound in the case $p = 5$ to $|A| = O\left(\frac{p^n}{\log\log n}\right)$, though our strategy seems unlikely to generalize to larger p.

Incidences between points and circles
(contact: cosmin.pohoata@emory.edu).

This project concerned the problem of establishing a nontrivial incidence theorem for points and unit circles in F_p^2.

In a remarkable paper from the early 2000s, Bourgain–Katz–Tao showed that

for any $\epsilon > 0$ and every set of n points P and n lines L in F_p^2 with $n \leq p^{2-\epsilon}$, there must exist $\delta = \delta($