L2 INVARIANTS AND THEIR RELATIVES FOR FINITELY GENERATED GROUPS

organized by
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Workshop Summary

1 The purpose of the workshop

The workshop bringed together experts and young researchers in the following fields: asymptotic group theory, ergodic theory, L^2-cohomology, geometric group theory, percolation, 3-manifold theory, analytic graph theory, von Neumann algebras and free probability. The common object of interest is a finitely generated group, sometimes with added natural conditions, like having a finite presentation. Some examples where ideas from one subject were applied in a ground-breaking way in another are: Gaboriau's work on distinguishing ergodic equivalence re- lations by developing a measure-theoretic analogue of l^2-Betti numbers, Lück approximation, and the solution of the measurable version of von Neumann's problem using percolation on transitive graphs by Gaboriau and Lyons. The meeting brought new understanding of the already existing directions and will further pave the way to building a general theory.

Each day at the workshop consisted of two talks in the morning introducing topics to be discussed in the afternoon, group work in the afternoon, reports from the groups, and evening talks.

In the next sections we shall briefly describe the morning and evening talks and then some of the main results of the afternoon activities.

2 Thom and Gaboriau

The workshop started with two introductory talks by A. Thom (Leipzig) and D. Gaboriau (ENS, Lyon). They introduced the main definitions and connections with measured equivalence relations and their graphings and the cost of actions on measure spaces.

3 Lück

W. Lück (Bohn) talked about approximations of L^2 invariants by finite structures. One of the earliest theorems of this kind (due to him) express L^2 Betti numbers of the universal cover of a manifold with residually finite fundamental group as limits of normalized Betti numbers of its covers corresponding to the finite quotients of the fundamental groups. This result follows from the analysis of the edge behavior of spectra of non-negative integer matrices. Extending such results to "higher-order" invariants (such as L^2 torsion) requires better control of this spectral behavior and is related to a number of conjectures, such as the determinant conjecture.

4 Lyons

R. Lyons (U. of Indiana) talked about application of the theory of percolation on infinite graphs to the theory of cost of measured actions.

5 Monod

N. Monod (Lausanne) explained the solution of Pisier's problem: existence of an example of periodic non-unitarizable finitely generated group. He explained why positivity of the first l_2-Betti number implies non-unitarizability. The proof is somewhat similar to the Pisier's direct proof of non-unitarizability of free non-amenable groups (presented by him during the previous AIM workshop). Later, during the afternoon group meetings, Osin explained how to construct a residually finite torsion group with positive first l_2-Betti number.

6 Biringer

Ian Biringer (Gibbs Assistant Professor, Yale) talked about asymptotic behavior of the Betti numbers of higher rank locally symmetric spaces as their volumes tend to infinity. The main result is a uniform version of the Lück Approximation Theorem.

The basic idea was to adapt the theory of local convergence, originally introduced for sequences of graphs of bounded degree by Benjamimi and Schramm, to sequences of Riemannian manifolds. Using rigidity theory we are able to show that when the volume tends to infinity, the manifolds locally converge to the universal cover in a sufficiently strong manner that allows us to derive the convergence of the normalized Betti numbers.

7 Peterson

J. Peterson (Assistant Professor, Vanderbilt) spoke of the proof of the following result: if a group Γ has a non-trivial cocycle into a weakly mixing unitary representation, then the group cannot be measurably equivalent to a direct product of two infinite groups. This generalizes similar results obtained for groups having positive first L^2 Betti number by D. Gaboriau. The proof actually works for von Neumann algebras (and was first obtained in that context) and Peterson has presented an adaptation of that proof written purely in the language of ergodic theory.

8 Kassabov and Ershov

M. Kassabov (Southampton, UK and Cornell) and M. Ershov (U. Virginia) were talking about constructing residually finite groups with "extreme" properties. Kassabov gave an example of a profinite (compact) group with two dense finitely generated subgroups one of which has property (T) and another one is amenable). Ershov explained how to use Golod-Shafarevich construction and its generalization to built a periodic infinite residually (finite p-) group such that every proper finitely generated subgroup is either finite or of finite index. One can assume, in addition, that the group is not amenable and even has property (T). Pisier noticed that this group could be a counterexample to the Dixmier unitarizability problem. In any case representation theory of Kassabov's and Ershov's groups is an interesting problem.

9 Medynets

K. Medynets (a postdoc in U. Ohio) gave two evening talks (total of 3 hours) about his and Grigorchuk's recent solution of a well known problem: they constructed a finitely generated infinite simple amenable group. The method of the construction is quite original and unexpected. They use the dynamics properties of subshifts and the so-called topological full groups of these dynamical systems (which have not been used in group theory before). It turned out that if the subshift is minimal and the alphabet is finite, then the topological full group is finitely generated, simple and amenable. After the talks Shlyakhtenko conjectured that one can start with a minimal action of any finitely generated amenable group, and use the uniqueness of its sofic approximation to prove that the correspomding topological full group is amenable. Ozawa and Monod noticed that if one starts with the R. Thompson group F acting on the cantor set, then the topological full group contains the R. Thompson group V which is not amenable. Thus if Shlyahtenko's conjecture is true, R. Thompson's group F is not amenable. The topological full groups are very interesting new objects in group theory and many natural questions about them (say, the dimension growth) have been proposed by Sapir.

10 Okun and Mineyev

B. Okun (U. Wisconsin at Milwaukee) talked about the Atiyah's conjecture which in some sense motivated the original interest in l_2-Betti numbers. He proved that the Atiyah's conjecture holds for right angled Artin and Coxeter groups. I. Mineyev (University of Illinois at Urbana-Champaign) explained his recent solution of the famous 40-years old Hanna Neumann conjecture, its reformulation in terms of l_2-Betti numbers, its generalizations and possible applications to the Atiya's conjecture. He introduced a new notion of the deep fall property which is the crucial ingredient in his proof of the Hanna Neumann's conjecture, and conjectured that an analog of Hanna Neumann conjecture and the Atiya's conjecture are true for groups satisfying that property. During his talk a potentially very fruitful connection with graphings appeared. In particular his choice of a subforest of tree covering turned out to be very similar to the construction of free forests in Lyon's talk. The only difference is that Mineyev's forests are labled by elements of an orderable group and Lyon's forests come from percolation and are labeled by real numbers.

11 Problem solving in groups

The first afternoon was devoted to a problem session. Several problems has been recorded which allowed us to plan some afternoon group work for the next three days. The problems were recorded by K. Juschenko (Lausanne) and have appeared on the Web site of AIM institute.

Several afternoon groups were devoted to problems that appeared in the first day, but some groups formed to discuss problems that arose during the morning talks.

Incremental advances have been achieved on several of the problems. This included extension of existing non-unitarizability results to larger classes of examples and progress on the question of the possibility of topological equivalence of actions of free non-abelian groups of different ranks.