Latinx mathematicians network III

organized by
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Workshop Summary

This is a brief report on the opening workshop for our AIM research community dedicated to young mathematicians of the Latinx/hispanic community who convened to work on 5 research projects/areas. The 60 participants included graduate students, postdocs, and a few young assistant professors (pre-tenure). They came from a variety of academic institutions in the USA and abroad. Our set up was unusual in two significant ways: First, this event happened during the COVID-19 crisis and thus had to be completely online which presented a challenge. Also, unlike other research networks at AIM that research on a single common topic, this network had five research topics. The glueing unifying force to the entire group was the intention to mentor and support the careers of this group of young mathematicians in the service of the Latinx/hispanic community. Their success means a measurable contribution against the blatant lack of representation of Latinx in the mathematical sciences. The schedule of a typical day was research during the morning, this was done in five separate teams, with a break for lunch and then, in the afternoon, the entire community gathered for professional development discussions. Topics included writing grants, organizing and setting up a research agenda, and life-work balance. The plan is for one year we will continue regular research work but once a month we will all meet again to report to the entire group and discuss more topics of professional growth. The first monthly meeting (in Sococo) will take place in September, 2021.

Finally, this community is the realization of many years of hard work on creating a community for Latinx mathematicians. We think that AIM was a great setting for this meeting and the creation of this community. We thank AIM for all the support and help we received and look forward to this year of innovative research. *Gracias!*

Results from the Five Research Teams

Codes from Geometry: This group began working on the construction of locally recoverable codes using ideas and tools from algebraic geometry over finite fields. Much of the week was spent getting the team up to speed on the fundamentals of coding theory, as well as getting comfortable with constructing codes experimentally using MAGMA. They worked through part of the seminal paper "A family of optimal locally recoverable codes" by Tamo and Barg. The original constructions given by Tamo and Barg have an elementary description, but can be re-interpreted as codes on a plane curve. We organically split into two groups: one group investigated new choices of plane curves to construct codes; the other group investigated what would happen if one got rid of the constraint that evaluation points lie on a plane curve. They have already produced a modest supply of new optimal locally recoverable codes, with proofs of their efficacy. In the coming months we plan to continue developing the group's understanding of algebro-geometric codes, in order to re-interpret existing literature under this framework, with the goal of generalizing existing constructions of codes with high information rate, good minimum distance, and modest locality parameters.

Mathematics education: This group discussed topics related to educational research, embodied cognition, and interactions in the classroom. Students listened to presentations prepared by the facilitators and by other team members; they read and summarized research articles, and worked collaboratively to propose research ideas. Hortensia offered a rich set of records collected in a complex analysis class for designing a possible study. After various deliberations and discussions, that included availability by the facilitators, the group originally proposed a very interesting project that will take advantage of the records and agreed to a plan to conduct a study over the next months and submit a paper for publication for a top tiered journal in mathematics education.Our goal is to complete a draft to a high tiered journal (Journal of Mathematical Behavior) that might be ready for publication by May 2022.

Measuring Polytopes: This group was divided into two projects: First, The Merino-Welsh conjecture (1999) states that the Tutte polynomial $T(x, y)$ of any matroid M satisfies that $T(1, 1) \leq \max(T(2, 0), T(0, 2))$. This conjecture has been widely studied for graphs, where it has been proved in many important cases. There is less evidence for this conjecture for matroids that do not come from graphs. Our goal was to study the conjectures for matroids built from finite root systems – one of the few interesting infinite families where the Tutte polynomial can be computed. We learned about very recent results from Kung (from a couple of weeks before our meeting!) that imply the conjecture for finite root systems. The next goal is to test the conjecture on various families of matroids that can be built starting from these as building blocks. In the future, we are also interested in studying various other conjectures on Tutte polynomials in the special case of root systems, where the combinatorics of Coxeter groups provides additional useful structure.

The second project was about Geissinger (1981) introduced and Dobbertin (1985) described a polytope associated to a poset P, arising from the study of valuations of distributive lattices. In his foundational textbook Enumerative Combinatorics, Richard Stanley asked the open-ended question of obtaining a more detailed understanding of this family of polytopes. They studied the Dobbertin polytopes of various families of posets P, using combinatorial and computational analyses, eventually focusing on the family of zig-zag posets Z_n, which seemed to provide an appropriate level of complexity. We discovered that this family already has very interesting properties, some of which we were able to prove. Our main discovery was an elegant formula for the volume - and more generally the Ehrhart theory - of these polytopes. We were able to prove that the Dobbertin polytope of Z_n has a unimodular triangulation. It remains to understand the combinatorial structure of this triangulation, which we expect will give a proof of our conjecture. More generally, we believe that these techniques will work for any poset of height two? where the polytopes are already quite interesting. We anticipate connections with Karola Meszaros's 2009 work on triangulations of root polytopes and Kirillov's subdivision algebra. Once we fully understand this story for height 2 posets, we will move on to wider families of polytopes where we can discover positive/negative results on Dobbertin polytopes.

Data-Driven Models: This was the largest group of researchers at 15 members, which was then further subdivided into three projects: The first project investigated models for Herd Immunity in Heterogeneously Mixing Populations. This project is motivated by the known disparity of impact that COVID-19, where communities of color have been affected at higher numbers than other communities. In those minority communities there is a non-uniform access to vaccines and health care. While we often hear 70 percent vaccination leads to community immunity, this is not realistic. What are the impacts of heterogeneity on thresholds for herd immunity? Does including racial/ethnic heterogeneity lead to a more accurate R_0 value for black, Indigenous, and people of color (BIPOC) communities? Another more theoretical problem is what are the effects of periodic shocks (e.g., lockdowns) on the dynamics of the (differential equations) model? Establish conditions for the existence of periodic cycles Study the phase transition between the disease-free steady states and periodic cycles

A second team investigated Optimal Vaccination Strategies to Increase Equity. These team focused on developing an optimization model where there is a partition of the population into groups $G_1, G_2, \ldots, G_n$. Let $x_i \in [0,1]$ for $i = 1, 2, \ldots, n$ denote the fraction of available vaccines that group G_i receives. One investigates functions $f_i : [0,1]^n \to (0, \infty)$ that model the benefits for group G_i given that they receive x_i portion of the vaccinations available. over all choices of x_i subject to the constraints $x_i \in [0,1]$, $\sum_{i=1}^n x_i = 1$. Some fairness criteria needs to be included, but what is the correct model of fairness? (i.e., demographic parity, disparate impact, equality of odds, calibration) There is no correct definition of fairness and unfairness and mathematically it has been shown it is not possible to satisfy all the definitions of fairness at the same time. These two teams have already assembled some concrete data from Colorado's health department and includes racial/ethnic heterogeneity.

The last project is a bit more theoretical regard learning a Mixture of Distributions. The goal is, given a probability distribution F in d-dimensional Euclidean space which is a convex combination of *ground distributions* of known fixed type, i.e., $F = w_1F_1 + w_2F_2 + \cdots + w_kF_k$. We wish to recover recovering w_i and F_i from the sample points. The team started with the case when the ground distributions are nice such as Gaussians or Poisson distributions. We aim to answer How many samples are needed to recover the F_i, w_i? Can we determine the origin of a sample according to the component distributions. We considered challenging variants, such as when the number of ground distributions k is unknown and the data is noisy. The methods are algebraic because Many families of distributions are determined by their *moments*. Gaussians are an important example of this. If the moments are polynomial, we can pinpoint a convex combination using a finite number of moments using real algebraic geometry.

Finding patterns: A classical connexion between Graph Theory and Linear Algebra comes from the notion of incidence matrix. Given a graph $\mathcal{G}$, consisting a finite collections of vertices and some (undirected, without weights) edges connecting a pair of vertices. Then one defines a matrix M with an entry m_{ij} equal to 1 if there is an edge connecting the vertices i and j. Loops, that is when the edge connects a vertex to itself, are allowed. Powers of the incidence matrix have a graph-theoretical interpretation: the entries of M^k count the number of paths of length k in $\mathcal{G}$.

A new form of powers of a matrix A was introduced by starting with a basis $\{v_1, \ldots, v_n\}$ and forming symmetric sums of tensor products of the vectors $\{Av_{\sigma_j}\}$, where σ runs over permutations in the symmetric group. This is called the `symmetric power of the matrix` A. The basic question discussed by the group is this: *assume A is the incidence matrix of a graph $\mathcal{G}$ what are the properties of the graph produced by the symmetric powers of A.* We denote this by $\mathcal{G}^{\mathrm{Sym}_k}$. The main accomplishment of the first week is the creation of a `Sage code` that produces such a graph. This code has given us the capability of testing a variety of conjectures. The remarkable fact is that even the most basic questions remain to be established. For example, if $\mathcal{G}$ is a connected graph, what is the number of connected components of $\mathcal{G}^{\mathrm{Sym}_k}$. We proved that if A is invertible, so are all their symmetric powers. We can also prove that a graph $\mathcal{G}$ embeds into its symmetric powers. We have observed that, by taking symmetric powers of a graph, one can produce loops. Why and how this occurs remains to be explained.

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