

# EMERGING APPLICATIONS OF MEASURE RIGIDITY

organized by

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## Workshop Summary

The American Institute of Mathematics held a five-day workshop on emerging applications of measure rigidity during June 13 to June 18, 2004, organized by Anatole Katok, Elon Lindenstrauss and Ralf Spatzier. The meeting was attended by 32 mathematicians, combining an attendance by researchers in dynamical systems, analytic number theory and spectral theory. A main goal of this workshop was to encourage interaction between researchers in these mathematical disciplines; we believe that this aim has been fully achieved.

The meeting had three different components to its setup: survey lectures, research talks and problems seminars. The number of talks was limited, and ample time was given to informal discussion. There were also a few additional evening talks expanding on the topics discussed in the survey talks and others.

The surveys were the core of the meeting, allowing the researchers with rather different backgrounds to learn about new tools and topics. This was crucial as mathematicians with very different core interests attended the meeting. Five series of survey lectures and eight research talks were given. Of particular note:

- (1) M. Einsiedler gave a detailed and very clear exposition of recent progress in classifying invariant measures under diagonalizable multiparameter actions, with an emphasis towards applications in Diophantine approximations and analytic number theory. The results presented are very recent, and the workshop was the first place the techniques were introduced in a detailed technical level to the experts.
- (2) A. Yafaev presented the Andre Oort conjecture, the necessary mathematical background, and the state-of-the-art towards this conjecture. This included a brief treatment of recent work by Ullmo and Clozel relating invariance of this conjecture to Ratner's theorem on dynamical properties of unipotent flows. It is quite possible that recent progress in understanding multiparameter or diagonalizable flows can be used for this problem.
- (3) D. Damjanovic presented very recent work introducing the classical techniques of KAM to the study of rigidity of diagonalizable multiparameter actions. Her joint work with A. Katok gives the best known results for local rigidity of partially hyperbolic actions of higher rank abelian groups on tori. Their methods may also be applicable to other homogeneous actions. This was much discussed informally during the meeting, and also in the problem sessions.
- (4) In a series of talks recent progress on quantum chaos was presented. J. Marklof gave a general introduction to quantum unique ergodicity and the results of Shnirelman, Zelditch and de Verdière as well as the phenomena of scarring and bouncing ball modes. E. Lindenstrauss explained his recent work on quantum unique ergodicity

for arithmetic surfaces which uses ideas from measure rigidity and in particular Ratner's theorem. A. Venkatesh and L. Silberman in two talks explained their work on arithmetic quantum unique ergodicity for higher rank locally symmetric spaces of non-positive curvature. Various problems and directions for research were already introduced in the talks.

- (5) P. Michel presented several equidistribution problems from analytic number theory related to the dynamical rigidity techniques, as well as pointed out the relevance of works of Y. Linnik.
- (6) G. Margulis presented new and exciting techniques which should allow effective quantitative version of the results regarding equidistribution of closed orbits of big groups for which only a noneffective qualitative proof were previously known.

The research talks discussed further advances in these areas and also in equidistribution and global rigidity.

We held two productive and lively problem sessions. The first was held in an open, unstructured, format; the second with a moderator, Anatole Katok, and preassigned problem posers. We found both formats useful and productive. The styles naturally complemented each other.

The problems proposed range in a wide variety of subjects: local and global rigidity, measure rigidity, equidistribution, diophantine analysis, quantum chaos, polygonal billiards divergent trajectories and other miscellaneous items. A careful write-up of these problems has been prepared by Alexander Gorodnik.