Topology and geometry of the moduli space of curves

organized by
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Workshop Summary

Goals and Events

The aim of the workshop was to bring mathematicians together for an active exchange of results, techniques and ideas on the geometry and topology on the moduli spaces of curves. This vibrant area is studied by topologists, algebraic geometers, and others. Because of the wide variety of tools used, it is essential that researchers have some familiarity with the ideas of other fields. However, those studying this important object have never met as a group before this conference.

To facilitate this exchange and create a working relationship between these various research groups, especially including topology and algebraic geometry, the workshop was loosely structured with two series of three lectures each where experts of one group gave an overview of the current state of the art in their field. On the first day the participants split in three groups (topologists, algebraic geometers, and those not in either camp) identifying topics worth further explanation. Many of these were then covered by the official lectures, and several more were addressed in extra sessions on Tuesday and Wednesday. An ad hoc session was organized on Wednesday evening by Dennis Sullivan, in the form of informal presentations and group discussion.

Typically, people in the audience were familiar with other members work, but had not met them personally before the conference. Despite the size of the workshop (there were about 40 participants with often as many as 50 people attending lectures) there was a good community spirit, often leading to lively discussion.

Mathematical Content

The workshop had several mathematical high points. We list a few:

1) Madsen and Galatius announced a generalization of the Madsen-Weiss theorem from two to any number of dimensions (joint work with Tillman and Weiss). This sheds new light of the on the proof of the Mumford conjecture on the stable cohomology of moduli space.

2) Mirzakhani presented her beautiful new proof of the Witten conjecture (first proved by Kontsevich in 1992) by explicitly integrating certain geometric functions over the moduli space, using hyperbolic techniques.

3) Faber lectured on the tautological ring, including his and Pandharipandes simple characterization.

4) Costello presented his view on Segals axiomatic TCFT which in particular expresses the Deligne-Mumford compactified moduli space as a functor of the open moduli spaces, and the closed string theory associated to an open string theory as the Hochschild homology of the latter. This in turn was used in a speculative lecture by Teleman to suggest that

the solution to the Mumford conjecture implies a structure theory for semi-simple TCFTs (including an extension to the Deligne-Mumford compactification).

Future Directions

A unifying theme throughout the conference was that of string and quantum field theory. Dennis Sullivan elegantly summarized this in his last talk. Mirror symmetry provides a dictionary between the symplectic topology and the holomorphic topology set-up of open/closed string theory. Both symplectic and holomorphic topology are difficult categories to work in, but algebra and differential topology provide a more amenable environment. The analogue of both open/closed string theories exist in these categories, and despite the necessary loss of information, results proved here help to explain and contribute to the original theories. Much work remains to be done to complete the picture in each category, and the crucial transfer of results between them has only just begun.

This workshop has been extremely helpful in fostering interaction between those working on similar objects and problems in these different categories. Another conference along similar lines might be appropriate in three to five years.

Further Comments

Extensive material for a website has been collected by graduate scribes Giansiracusaand and Maulik including an annotated list of references, a glossary, list of questions and problems, as well as notes for several lectures.

In the wrap-up session, after a suggestion from R. Cohen, the group expressed interest in an e-mail discussion list consisting initially of the workshop participants.

The problem session was felt not to have been very successful. In part this was certainly due to the fatigue of the participants and to the earlier similarly-spirited session on Monday. But the combination of the narrowly defined focus and heterogeneous audience seemed less conducive to a problem session than a typical AIM conference. The bigger questions had already been addressed in lectures, while more detailed questions tended to be less transdisciplinary.

All in all we believe this was a very successful conference. We would like to thank AIM and the NSF for their generous financial and organizational support.