Nonlocal differential equations in collective behavior

organized by
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Workshop Summary

Overview. The workshop focused on the mathematical theory of nonlinear integro-differential equations appearing in the context of collective behavior of interacting agents. One of the purposes of the workshop was to give an overview of several fields of application that use similar techniques, so that researchers are aware of neighboring areas that may benefit from the same kind of ideas. The main topics treated were (i) the Landau equation from plasma physics, both from theoretical and numerical standpoints; (ii) aggregation-diffusion equations and related problems; (iii) new links between nonlocal partial differential equations (PDEs) and machine learning algorithms; and (iv) recent gradient flow techniques for nonlocal PDEs.

We had participants from universities in the United States, Canada, United Kingdom, Spain, Austria, Germany and Italy, including both leading experts and a good number of early-stage researchers. Most of the participants knew very well the theory of nonlocal PDEs, and a few were working on more applied problems for which techniques in nonlocal PDEs were relevant.

Summary of talks. The aim of the talks at the workshop was to give an overview of a particular subfield, rather than to present advances in a specific aspect of a problem. This was well-received; people mentioned that it was very useful to have introductions for topics outside one's immediate expertise. On the first day, Maria Gualdani and Stanley Snelson gave an overview of the state of the art on the mathematical theory of the Landau equation, and Yao Yao gave a review of the aggregation-diffusion equation and the open problems regarding its behavior. On Tuesday, Jacob Bedrossian presented the recent ideas on Landau damping for the Vlasov-Poisson equation and fluid-mechanics models; Razvan Fetecau presented recent work on the aggregation equation on manifolds. On Wednesday, Li Wang and Jingwei Hu reviewed numerical analysis techniques for nonlocal equations, and specifically for rarefied gas and plasma physics models. Dejan Slepčev presented recently discovered links between nonlocal PDEs and unsupervised machine learning mathematical models. On Thursday Katy Craig gave a talk on aggregation-diffusion equations, their discrete approximations and recent numerical methods. Michael Levy gave a brief talk on the modeling of bacterial aggregation, and Andre Schlichting gave a review of recently developed gradient flow methods that apply to both continuous and discrete nonlocal problems. On Friday, Nancy Rodriguez presented some nonlocal population models of reaction-diffusion type with applications in ecology modeling, and Nicola Zamponi gave a talk on systems of nonlinear diffusions with cross diffusivity.

Problems discussed in work groups. A total of 13 problems were selected after the open discussion on the first day of the workshop, and participants later chose to take part in one of these groups. After the first day (with one exception) the groups stayed the same for the entire duration of the workshop.

One group worked on the aggregation equation on manifolds, focusing at first on issues related to well-posedness of solutions. This question is particularly interesting if the distance on the manifold is considered. This group later split and changed for two days to the topic of discrete aggregation equations, which arise in connection to machine learning problems. Some ideas were proposed in both topics, with participants agreeing to follow up on this after the workshop.

Two groups were discussing the Landau equation. One focused on numerical methods and proposed a modification of existing blob methods that may be useful for the Landau equation. The other group discussed theoretical issues and focused in particular on possible simplifications for the Landau equation and classification of blow-up profiles.

A group studying the geographic distribution of gangs proposed some simplifications and gave preliminary analytical results.

The group studying the aggregation-diffusion focused on the issues related to the behavior of solutions when the potential is smooth and attractive and the diffusion is linear. Some similarities were observed in problems in statistical mechanics where the escape time from a potential well is estimated.

Concluding remarks. In our opinion the workshop was extremely well-received, with many positive feedbacks from the participants. Many participants were eager to continue discussions on the problems after the workshop, and several new connections were pointed out that may lead to new collaborations and organization of further research and educational activities.