

PAINLEVE EQUATIONS AND THEIR APPLICATIONS

organized by

Estelle Basor, Yang Chen, and Michael Rubinstein

Workshop Summary

The goal of the workshop was to bring together experts in random matrix theory and number theory to study the role of Painleve equations in number theory, random matrix theory, and the many interesting connections between the two theories.

Random matrix theory has provided a rich set of statistical results and tools for analyzing the eigenvalues and value distribution of characteristic polynomials of Hermitian matrices and related statistics of matrices in the classical compact groups and the connections to Painleve equations in many settings is now well understood.

It is believed that several problems in number theory also have behaviors that are governed by Painleve equations. These include the asymptotics of the moments of the derivative of the Riemann zeta function, mixed moments of the zeta function and its derivative, and in the moments of the average of the k -th divisor function in short intervals.

Other questions of interest involve describing statistical quantities that exist more directly in random matrix theory at the level of finite, fixed size matrices. For example, finding expressions for determinants of non-classical Hankel matrices that depend on parameters, (or with perturbed weights).

All of the above questions were discussed at the workshop. The morning talks summarized basic facts about Painleve equations, illustrated basic quantities of interest in number theory, and gave derivations of the equations in certain specific problems. In the afternoon, the participants broke into four groups. The four groups tackled a variety of questions and are briefly described below.

One group concentrated on deriving formulae for mixed moments, averaged over the circular ensemble, of the characteristic polynomial and its derivative, and using these to predict similar formulae for the zeta function. They also considered the problem of the moments of the logarithmic derivative of the characteristic polynomial finding connections to Painleve equations and integrable systems.

Another group considered the expression for the leading term of the moment of the derivative of the characteristic polynomial for the circular unitary ensemble and how to extend the answer to non-integer values. This group also investigated the mixed product situation with one of the indices equal to $1/2$.

The third working group studied the problem of the location of the zeros and boundaries of Painleve polynomials, that is, polynomial solutions to the various Painleve equations.

The last group was concerned with some questions that involve the β ensembles and also considered the problem of Hankel determinants with indices of the form $2i + j$.

Many partial results were obtained and it is hoped much work will continue after the workshop.