Phase Transitions

organized by
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Workshop Summary

This workshop was a truly interdisciplinary effort, bringing together physicists, chemists, computer scientists, and mathematicians to discuss a single, focused topic: Are there useful parallels between the classical phase transitions of statistical mechanics (ice to water to steam) and the phase transitions of computer science (3-satisfiability problems), combinatorics (birth of the giant component in random graphs, and probability (cut-off phenomena in mixing of Markov chains).

There were a number of last-minute changes and unexpected cancellations. But the physicist Michael Fisher stepped in and was terrifically useful. He is one of the world's great statistical mechanicians; he arrived early and worked late. He is such a stellar figure that it made the rest of us think we were part of an historical event.

Because of the shift in personnel, we cut out combinatorics (although Yuval Peres gave a useful tutorial on phase transitions in random graphs).

Every day began with two expository tutorials in the morning, working groups in the early afternoon, and a round up at the end of the day. Our first day had a broad overview of phase transitions in physics (Tom Witten) and a tutorial on rapidly mixing Markov chains and the cut-off phenomenon (Persi Diaconis). The non-physicists present were amazed at the rich variety of special vocabulary used to describe a huge variety of phases. This was unpacked continuously during the conference. It will lead to a definition of "phase," "phase transition," and "order parameter" to be posted on our conference Web site (Charles Radin, Michael Fisher, and Hans Anderson are arguing this out as the present account is being written).

For the first day, the participants were assigned to groups in a balanced way and asked to discuss and report on the question, "Are there useful connections between the two areas (phase transitions in physics and card shuffling)? This question developed as a running theme during the conference. Most participants felt there wasn't. Despite the similarity, small changes in the number of shuffles lead to large changes in the degree of mixing. Many phase transitions in physics are reversible—you can unfreeze ice, while we can't unmix cards. Also, physical phase transitions are characterized by an observable "order parameter," while the observable feature that signals an unmixed deck (number of fixed points) was deemed "non physical" by the majority.

The report above, a brief summary of one feature of the working group's work throughout the week, led to some new mathematics. Determined to show the physicists that fixed points serve as an order parameter, Diaconis (working with Jason Fulman) managed to prove that, near the cut-off point, $\frac{1}{2}$ log n, the conditional probability of the arrangement, given the number of fixed points, is uniform to good approximation. This was a hotly debated topic during the meeting. An earlier claim by Diaconis (that the above was true after n shuffles) was shot down during the meeting (by Boris Pittel, Elchanan Mossel, and Peres) using the translation of random transpositions shuffling into the language of the evolution of the random graph. This appearance of order parameters as a topic of study in Markov chains is a contribution from our meeting.

On day two, we had an introduction to non-equilibrium phase transitions (Sue Coppersmith) and to transitions in k-SAT problems (Peres). Most of us were amazed at the heated arguments around the question, "Is glass a liquid?". One of the most surprising things this scribe learned was the following: one argument for treating glass as a liquid is that old church windows are thicker at the bottom. Shockingly, it seems they were made that way, originally. To observe that amount of flow would take hundreds of thousands of years.

A highlight of the afternoon's session was Radin's demonstration of an old Indian magic trick—rice packed in a jar, a knife poked in, and the knife pulled up carries the rice and jar with it. Radin has just finished a sequence of experiments on "packing phase transitions" (with the Harry Swinney group at Austin). A number of working groups discussed the exact experimental conditions. There was sufficient interest in this topic that two further survey talks were added. One on "Phase Transitions and Jamming" and one on "Droplet Breaking." These were wonderful talks with amazing demonstrations and additional rich vocabulary to illuminate the observed physics.

There was not quite as much interaction among the computer science theorists and the topics above as we hoped for. Still, the CS theorists clustered together and worked at a list of problems they thought important. These, and some problems developed by the other groups, were carefully worked over during the week. They are already posted on the Web site.

The organizers found the conference surprisingly successful. People really did talk across fields, exchange literature, and work through problems until they were clear to all sides. It is hard to keep track of activity afterward, but each of us has had some real follow up.

Two demographic points:

- Women were unusually well represented, both in numbers and in their contributions.
- There were a number of graduate students who drifted in from Stanford and Berkeley. They seemed lost and a mentoring scheme might have helped.