

$$M = RS, \quad K \text{ cpt}, \quad G = K^{\mathbb{C}}$$

$$A(g) = \frac{1}{2} \int \langle g^{-1} dg \wedge * \bar{g}^{-1} d\bar{g} \rangle$$

Harm.  
maps:

$$2(g^{-1} \bar{\partial} g) + \bar{\partial}(g^{-1} \partial g) = 0 \quad \text{Why compl. soln?}$$

$$0 \neq \lambda \in \mathbb{C}$$

$$A^\lambda = \frac{1}{2} \left(1 - \frac{1}{\lambda}\right) g^{-1} \frac{\partial g}{\partial z} dz + \frac{1}{2} \left(1 - \lambda\right) g^{-1} \frac{\partial g}{\partial \bar{z}} d\bar{z}$$

$$\lambda = 1: \text{trivial} \quad \lambda = -1: g^{-1} dg$$

Th. 1  $g$  harmonic  $\iff A^\lambda$  flat  $\forall \lambda \neq 0$ : loop in space of fl. conn.

Pohl-Meier, Zakharov —

$$dA^\lambda + \frac{1}{2} [A^\lambda \wedge A^\lambda] = 0$$

Harm. map  $\iff * (g^{-1} \partial g)$  is flat

0 curvature repr.

Uhlenbeck / G. Segal:

For simplicity  $M = S^2$

(Uhlenbeck)

Def An extended soln is a map  $E(z, \lambda) \in G$

s.t. 1/ holo in  $\lambda$

2/ reality condition:  $|\lambda| = 1 \implies E \in K$

$$E^{-1} dE = \frac{1}{2} \left( (1 - \lambda^{-1}) g^{-1} g_z dz + \lambda (1 - \lambda) \bar{g}^{-1} g_{\bar{z}} d\bar{z} \right)$$

$$\text{Map}(\mathbb{C} \setminus \{0\}) \rightarrow G$$

{Ext solns}  
 $\downarrow$   
 Harm  $(M, K)$

$E(z, \lambda)$   
 $\downarrow$   
 $g(z) = E(z, \lambda = -1)$

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$\{k \in LK\} \rightarrow \text{Ext solns}$

$k: \mathbb{C} \xrightarrow{\text{holo}} G: \text{Harm}$

$(\lambda) = [\Rightarrow k(\lambda) \in K$

$$\begin{pmatrix} a & bz \\ -b\bar{z} & \bar{a} \end{pmatrix}$$

$z \mapsto E(z, \lambda)$

Ext. soln:  $E: M \rightarrow LK = \{h: \mathbb{C}^n \xrightarrow{\text{holo}} G \mid h(S^1) \subseteq K\}$

Harm. maps  $g: M \rightarrow K$   $h(1) = 1_K$

(Segal)

Pressley-Segal:  $LK$  is a cpx mfd

$$K/T \cong G/B^+$$

$$LK_K \cong LK \cong LG/L^+G \quad \text{holo in disk}$$

Factoris<sup>n</sup> result: Gram-Schmidt

$$g \in LG \Rightarrow g(\lambda) = \prod_{LK} k(\lambda) g_+(\lambda)$$

$\uparrow$  holo in  $\Delta = \{\lambda \mid |\lambda| < 1\}$   
 $L^+G$

Action of LG on space of harm. maps

Bäcklund x fns

$LG \times \mathbb{R}^k \hookrightarrow$

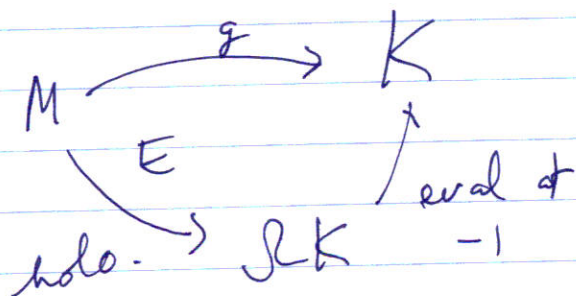
$(h(\lambda), k(\lambda))$   
 factoriz<sup>n</sup> unobstructed: cf. KAN, Iwasawa

— Factoriz<sup>n</sup> depends ctsly on ...

Th  $\exists$  action  
 $LG \times \{\text{extended solns}\} \hookrightarrow$

$h(\lambda), E(z, \lambda) \rightarrow (z \mapsto h E(z, \lambda))_{\mathbb{R}^k}$

$\mathbb{T}^*_{<1}$  act (Morse fn is energy fn on  $\mathbb{R}^k$ )



Twistor constr. applied to harm. maps.

Choose ext soln to go into Schubert vty

Bott-Samelson: A Schubert vty is iterated sphere bundle



Poisson Lie gp str for LG

(4)

Yangian (?) - gm gp analogue

Senenkov - T-S late 80s

$\frac{1}{2}$  eval at  $\lambda \neq -1$  get soln to WZW

WZW:  $\frac{1}{1} \text{tr}(g) + 2\pi i k \text{WZW}(g)$  critical  $\frac{1}{1} \sim k$   
action

$$\left(\frac{1}{1} + k\right) \text{tr}(g^{-1} \partial g) + \left(\frac{1}{1} - k\right) \text{tr}(g^{-1} \bar{\partial} g)$$