Bounded gaps between primes

organized by
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Workshop Summary

The workshop was devoted to the recent ground-breaking developments concerning the existence, for each fixed k, of a finite bound B(k) for which one can find arbitrarily large x such that the interval [x, x+B(k)] contains at least k prime numbers. In case k=2, this is due to Y. Zhang, and for arbitrary k, to J. Maynard and independently T. Tao. All three of these gave talks at the workshop. These results had already spurred a number of further developments and the talks by Tao (on work of D. Polymath), and of Maynard (on large gaps between primes) exemplified this. Zhang had talked on his vital improvement of the Bombieri-Fouvry-Friedlander-Iwaniec extensions of the Bombieri-Vinogradov theorem, which had been the main ingredient in his breakthrough result.

The first talk, given by S. Graham, served as an introduction, at first to the classical Selberg sieve weights, the work of Goldston- Pintz-Yildirim on small gaps between primes on which everything else depended, and then to the multi-dimensional Selberg weights (briefly mentioned much earlier by Selberg, but never having found much use), which provided the main new ingredient in the work of Maynard and of Tao.

Subsequent lectures, given by J. Pintz, P. Pollack, L. Thompson, T. Freiberg and J. Thorner were concerned with the speaker's recent development of these methods to successfuly attack another problem in the area (in the lectures of Pintz and Pollack, several results were described). The final lecture, given by A. Harper dealt with a rather different topic, his beautiful recent work on exponential sums over smooth numbers.

In the usual AIM format, the afternoon sessions consisted of small group discussions of relevant topics, hopefully ripe for progress, spurred by the above works. Some topics discussed were:

i) on the distribution of zeros of L-functions, for example the potential for using multidimensional Selberg weights, by analogy to the way Selberg had used his one dimensional weights to study the the zeta and L-functions, before he ever used them for the sieve.

ii) Gaps between E_k numbers (that is integers with exactly k prime factors).

iii) bounded (and/or large) gaps between Piatetski-Shapiro primes.

iv) sieving limits and the parity problem (a nice characterization of this was found during the workshop).

Much good discussion occurred, some progress was made, and we feel that, at the very least, everyone went home with a stock of interesting problems to think about, which was larger than the one they had on arrival.