The property of rapid decay

organized by
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Workshop Summary

This workshop was devoted to the property of Rapid Decay for cocompact lattices in semisimple Lie groups. The property of Rapid Decay (property RD) concerns convolution operators and captures certain aspects of the asymptotic geometry of the underlying group. Valette's conjecture that all cocompact lattices in any semisimple Lie group have property RD is still open in many cases and was the main focus of this workshop.

Initial lectures by *Michael Cowling* and *Alain Valette* were devoted respectively to the basics of property RD and to its relevance in various areas of mathematics. *Dave Morris* who, at the time, was not yet familiar with property RD, gave a lecture presenting general facts of a geometric and algebraic nature concerning cocompact lattices in higher rank simple Lie groups. Later in the week, *Paul Jolissaint* explained a proof of property RD for word hyperbolic groups. Those lectures were aimed at people unfamiliar with property RD and its applications, or people familiar only to certain specific aspects of it. During the week, *Indira Chatterji, Jacqui Ramagge* and *Mark Sapir* presented proofs that various classes of groups have property RD.

On the second day of the workshop, the participants started working on potential proofs for Valette's conjecture. *Talia Fernos* tried to attack the more general problem of showing that property RD is inherited by discrete cocompact subgroups. She showed to a subset of the participants where the first difficulties arise. *Mattia Perone* and *Romain Tessera* enthusiastically discussed this problem later in the week but, at the end, they didn't seem to overcome the difficulties.

Emmanuel Breuillard explained a promising approach that consists in trying to mimic classical proof of property RD for semisimple Lie groups. This aproach would use recent deep results on the equi-distribution of lattice points. Later in the week, *M. Cowling* explained to the audience some related earlier attempts (e.g., in joint work with Tim Steger).

Other discussions and works that emerged during the week include the following. *E. Breuillard* explained his conversations with *Anna Wienhard* —triggered by *Cowling*'s opening lecture— on an equivalent characterization of RD in terms of decay of matrix coefficients. This yields a natural way of defining property RD for arbitrary unitary representations and is pleasing to people who are familiar with representation theory. *D. Morris*, *J. Ramagge* and *A. Wienhard* worked hard on a special case of Valette's conjecture, the case of Sp4. This is the most interesting open case in some sense. *Cristina Antonescu*, *Paul Jolissaint* and *Corran Webster* discussed a strengthening of property RD which they call operator RD, abbreviated below to ORD. This gives a new insight on property RD and open a new direction of research. *Mark Sapir* described a group (considered by Dani Wise) which may be one of the simplest explicit group for which property RD has neither been proved or disproved.

A decision either way for that group would be interesting as it would either require new techniques or introduce some new obstruction.

To summarize, the participants responded enthusiastically to the various activities of the workshop. The presence of people with many different backgrounds helped everyone stay away from highly specialized discussions. Instead, basic ideas were discussed. The success of the workshop is threefold. First and formost, it introduced property RD to an enthusiatic group of young mathematicians who are well equipped to make contributions to this area of research. Second, it lead to useful exchanges of ideas between people who had done work on different aspects of property RD in the past. Third, it gave a renewed impetus to research on property RD and Valette's conjecture.

E. Breuillard is one of the young mathematician who was introduced to property RD at this workshop and became very interested in Valette's conjecture. He reports that he is working on several ideas but that things are more difficult than he first thought.

D. Morris, *J. Ramagge* and *A. Wienhard* report that they have made considerable progress in understanding the problem created by right angles in the case of Sp4. They do not know yet have a definitive result but are tracking down interesting features of the underlying geometry.

C. Antonescu, *P. Jolissaint* and *C. Webster* are actively exporing there ideas concerning operator RD.