

# RATIONAL SUBVARIETIES IN POSITIVE CHARACTERISTIC

organized by

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## Workshop Summary

This workshop was about studying rational curves and rationality questions in characteristic  $p$ . The goal was to bring together experts in characteristic  $p$  techniques together with experts in studying rational curves and see if the collaboration could yield new questions results and perspectives. The workshop followed the traditional AIM format of talks in the morning, followed by group sessions in the afternoon. The talks were intended to introduce the workshop participants to a broad array of techniques and problems in the area. The talks were as follows:

- Martin Olsson: Background on Trace Formulas and Point Counts
- Rachel Pries: Open Questions about Fundamental Groups of Curves and Moduli Spaces of Abelian Varieties in Positive Characteristic
- Matthew Woolf: Decomposition of the Diagonal and Beyond
- Qile Chen: Separable Rational Connectedness
- Hélène Esnault: Lefschetz Theorems
- Yiwei She: Shafarevich Conjecture
- Daniel Litt: Anabelian Aspects of Hyperbolicity
- Bianca Viray: Constructions of Severi-Brauer Bundles over, and Cyclic Covers of,  $\mathbb{P}^2$  with Brauer classes and Possible Extensions to  $\mathbb{P}^n$ ,  $n > 2$
- Eric Riedl: Rational Curves on Hypersurfaces
- Roya Beheshti: Rational Curves on Hypersurfaces II

The goal of the working groups was to get further knowledge about specific open problems in the area, and to begin work on solving these problems. Working groups worked along a variety of different avenues.

- One group focused on extending results on rational curves in characteristic 0 to positive characteristic. They started by considering the techniques of Harris-Roth-Starr, Beheshti-Kumar, and Riedl-Woolf, and preliminary results indicate that the techniques work in characteristic  $p$  as well.
- One group worked at understanding Brauer group computations as a way of obstructing irrationality. Bianca Viray gave a talk about computing Brauer groups and one of the working groups focused on trying to compute a more general class of examples of irrational varieties using these techniques. The group understood some of the limitations of Totaro's degeneration of varieties to a union of two reducible varieties and the various classes of decomposition of the diagonal.
- One group focused on studying rational subvarieties of the moduli space of rational curves and the moduli space of abelian varieties.

- One group focused on finding an odd-degree unirational parameterization of a cubic hypersurface in characteristic 2, which would imply that in characteristic 2 there is an integral decomposition of the diagonal. There was an idea using 2-torsion of certain elliptic curves that appeared quite promising, but after the workshop, the group found that the parameterization they were considering ended up having even degree after all.
- Some working groups worked on showing that hypersurfaces of large degree tend to contain few subvarieties in a fixed family. For instance, the group worked on a technique to show that in characteristic  $p$ , a hypersurface of sufficiently large degree contains no hyperelliptic curves. The technique uses decompositions of the diagonal to establish a base case and then uses a Grassmannian technique to establish nonexistence of rational curves. The main challenge is in working out base cases.
- One working group worked on extending results of Ein-Lazarsfeld-Ullery to characteristic  $p$ . They obtained some partial results but agreed that inseparable covers would complicate the picture.
- One working group worked on finding new obstructions to rationality in characteristic  $p$ . They focused on studying the higher cohomology of the structure sheaf and were able to find an obstruction, but working out nontrivial examples seems to be challenging so far.