Research experiences for undergraduate faculty

organized by
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Workshop Summary

On July 20-24, 2009, the American Institute of Mathematics (AIM), with support from the National Science Foundation (NSF), hosted the Research Experiences in the Mathematical Sciences for Undergraduate Faculty (REUF 2) workshop for faculty members from undergraduate colleges. The faculty participants in the REUF 2 workshop teach and mentor students who participate in REUs and/or apply to graduate schools in the mathematical sciences. The goal of the workshop was to introduce these undergraduate faculty members to current research projects in graph theory and algebra that are suitable for undergraduate research projects in mathematics. The activities of the workshop were:

- Introduction of problems and their contexts by the project leaders.
- Selection of problems to investigate by participants and organization of working groups for each problem.
- Mini-lectures on background information within groups .
- Work in groups on the problems.
- Group reports on progress.
- Presentation about using the free open-source computer mathematics system Sage.
- General discussion on designing good undergraduate research experiences.
- Follow-up planning.

There were twenty-four participants representing seventeen institutions. The workshop began with lectures on problems lead by Nathaniel Dean, Texas State University; Leslie Hogben, Iowa State University and AIM; Philip Kutzko, University of Iowa; and Kent Morrison, California Polytechnic State University. The projects that the participants selected and worked on are described briefly below; additional problems were also presented at the beginning of the workshop.

Research Problems

Sphere-of-Influence Graphs (Dean)

Sphere-of-influence graphs (or SIGs) are used in pattern recognition and computer vision as a primal sketch intended to capture the low level perceptual structure of visual scenes consisting of dot patterns. Let $S = \{x_1, x_2, \ldots, x_n\}$ be a finite set of points in the plane. For each point $x \varepsilon S$, let $\rho(x)$ be the distance to any nearest member of S, and let $C(x)$ be the circle of radius $\rho(x)$ centered at x. The SIG for S, written $G(S)$, is the graph with vertex set S where vertices x and y are adjacent if and only if the circles $C(x)$ and $C(y)$ intersect at least twice. If a graph G is isomorphic to $G(S)$ for some point set S, then we say that

G is a SIG. It is known that a SIG with n vertices has at most $17n$ edges. Further, all SIGs can be realized by a set of points with integer coordinates. This last fact suggests a graph invariant called the screen size of a SIG G defined as the smallest integer k that some set of points in a $k \times k$ integer lattice realizes G. The screen size is of importance because of its original application to pattern recognition and computer vision where (for example) we might be only interested in SIGs that have screen size smaller than 255. After learning the necessary background, this group determined tight bounds on the screen size for various graph families; work continues that may lead to publication of results.

Minimum Rank of a Graph (Hogben)

A simple graph can be used to describe the family of real symmetric matrices $A = [a_{ij}]$ having nonzero entry a_{ij} exactly when the edge ij is present in the graph. The minimum rank problem for a graph G asks us to determine the minimum $mr(G)$ of the ranks of the matrices described by G. The maximum nullity $M(G)$ is the maximum of the nullities of the matrices described by G; obviously $mr(G) + M(G) = |G|$, where $|G|$ is the number of vertices of G. The zero forcing number is a graph parameter that is an upper bound for maximum nullity. This is a very active area of research and some of the theoretical approaches would be difficult for undergraduates, but examples have played an important role in making and proving conjectures. Methods of construction of low rank matrices lead an upper bound for minimum rank and the zero forcing number provides a lower bound on minimum rank; these techniques can be understood by undergraduates and used to construct examples. AIM hosts an on-line catalog of minimum rank of families of graphs `http://aimath.org/pastworkshops/catalog2.html` in which such examples can be recorded where they will be available for in future research. After learning the necessary background, this group determined the minimum rank, maximum nullity, and zero forcing number for various graph families; work continues that may lead to publication of results.

Cyclotomy using Representation Theory (Kutzko)

The study of *cyclotomy* (which comes from the Greek words for“circle division”) goes back to Gauss, who was interested in determining which regular polygons could be constructed only using a straightedge and a compass. His idea was work with *nth* roots of unity (because these are the vertices of the unit circle) and to find polynomial equations that were satisfied by certain sums of these roots, called *periods*. These periods, and other quantities associated to them, have more recently shown up in coding theory, difference sets and other applications of mathematics. It turns out that one can construct finite groups whose character values are periods and that one can use character theory to study these periods and other quantities associated with them. This project began with an elementary introduction to the representation theory of finite groups and continued with the construction of groups that may be used in cyclotomy. This led to several specific computational problems and work continues that may lead to new results in this area.

Groups of Perfect Shuffle (Morrison)

Consider a deck of *52* playing cards. Divide the deck in half and shuffle the cards. A perfect shuffle is achieved when the cards are interlaced perfectly. The “in-shuffle” is obtained when the second card of the perfect shuffle is the same as the top card of the original arrangement of the deck. The “out-shuffle” is obtained when the top card of the perfect shuffle is the top card of the original arrangement of the deck. If we perform a sequence of “in-shuffles” and “out-shuffles”, we will obtain a permutation of the original arrangement of the deck. The set of all such arrangements forms a subgroup, $G_{2,52}$, of the symmetric S_{52}. We generalize this problem for a deck of kn cards with k piles. Divide the deck into k piles of equal size. Arrange the piles in some order from left to right. A perfect shuffle is an arrangement of the deck obtained by picking a card from each pile going from left to right, cycling back around to pick the next k cards, and continuing until all cards have been picked. We denote the group of all possible perfect shuffles of the deck by $G_{k,kn}$. This group is a subgroup of the symmetric group S_{kn}. The goal was to the determine the structure of $G_{k,kn}$, for various positive integers, k and n.

Group Discussions about Undergraduate Research

Attributes of a good Undergraduate Research Problem

- Long term projects that involve independent intensive learning and discovery on topics that are unknown to the students.
- Good projects are easily understood in the beginning of research. To attain such access, some projects may need to be preceded by a mini-course these are more suitable for an REU that can provide such.

Practical advice for a good Undergraduate Research Experience

- The advisor should be very interested in the project on which the student is working.
- Provide students with a variety of problems from which to choose.
- Occasionally, it is good for the students to see the project director fumbling through the mathematics.
- Do not avoid difficult projects. Allow the student to make incremental progress. First give the hard problem, and then give them small incremental problems. You want the students to have that wonderful feeling of solving problems.
- Occasionally, you may need to hold the students hands.
- If available, include graduate students in undergraduate projects.
- The students should meet with the advisor on a regular basis, regardless of whether progress has been made.
- Some students begin working on senior projects during their sophomore year. Consider having students work in a form that is accessible to freshmen and sophomores.
- Students should be expected to write a report, disseminate results through conferences and poster presentations. Students may write grant proposals. Students going on to graduate school should learn to use LaTeX; it is good to provide LaTeX source files from prior projects.
- Try to get a student in an REU that is related to their academic year project.
- The point of mathematics is to ask questions. Get the student to ask questions.

Organizations Relevant to Undergraduate Research/Undergraduate Faculty involvement in Research

Several participants are involved with various organizations in the mathematical sciences that have programs supporting undergraduate research, supporting faculty who are involved with undergraduate research, encouraging students to pursue graduate degrees in the mathematical sciences, and/or broadening participation in mathematics. This list (of organizations and URLs) contains only those mentioned by participants and is not intended as a complete list of such organizations.

- American Institute of Mathematics `http://www.aimath.org/`
- Career Mentoring Workshop (CaMeW) `http://www.wheatoncollege.edu/CaMeW/application.html`
- Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) `http://dimacs.rutgers.edu/`
- Enhancing Diversity in Graduate Education (EDGE) `http://www.edgeforwomen.org/`
- National Alliance for Doctoral Studies in the Mathematical Sciences (Alliance) `http://www.mathalliance.org/`
- National Association of Mathematicians (NAM) `http://www.Nam-math.org/`
- National Research Experience for Undergraduates Program (NREUP)/Strengthening Underrepresented Minority Mathematics Achievement (SUMMA) `http://www.maa.org/nreup/`
- Summer Program In Research And Learning at the University of Maryland (Math SPIRAL) `http://spiral.math.umd.edu/`
- Summer Undergraduate Mathematical Sciences Research Institute (SUMSRI) `http://www.units.muohio.edu/sumsri/`
- Texas Mathworks `http://www.txstate.edu/mathworks/`

Follow-up for REUF 2

Many participants plan to offer undergraduate research topics related to the problems presented in this workshop; others plan to apply techniques for enabling those new to an area to quickly begin research to other topics they are interested in offering undergraduates.

Several of the research groups made progress on publishable results, and some of the participants want to continue to do research on the topic presented. The participants of the groups of Sphere-of-Influence Graphs/Screen Size and Minimum Rank of Graphs are working to extend their results and write them up to submit for publication. Participants of the group on Cyclotomy using Representation Theory have made arrangements to meet this fall to continue their collaboration.

The principal activity to maintain the momentum of REUF 2 will be a reunion of REUF 2 participants at the Joint Mathematics Meetings (JMM) in San Francisco on January 12, 2010 (the day before JMM begins, although pre-meeting events are taking place then). Other types of activities to keep the participants and leaders connected were discussed during the workshop and include an e-mail list-serve, database for undergraduate reports, and continuation of the AIM participant page; any of these not already implemented by January 2010 will be discussed further at JMM.

Activities planned for the REUF 2 reunion at JMM include discussion of undergraduate research projects related to this workshop in which the participants have been involved, continuation of small group research for those interested, and discussion of future REUF workshops (including topics and types of follow-up). Partial funding for REUF 2 participants to attend this reunion is available from the NSF conference grant that supported REUF 2. Participants have been asked to notify the organizers of their plans to attend the REUF 2 reunion (and whether they need partial funding to do so) by the end of September.

Reflections for Future REUF workshops

This workshop is the second in a planned series. The first was Research Experiences in Linear Algebra and Number Theory for Undergraduate Faculty (RELANT, now abbreviated REUF 1), which was funded by AIM through the ARCC grant from NSF. In both workshops, participants expressed the view that the workshop was a great opportunity to learn about areas of research that are different from ones specialty or teaching assignments. The AIM-style workshop (where research is done collaboratively in small groups) was seen as ideal for this type of workshop.

Participants would also like to have workshops in other areas. Topics for future workshops that were suggested at REUF 2 included analysis, topology, and applied mathematics. Several participants with an applied math background expressed a strong preference for projects in applied mathematics. However, others felt that projects in pure mathematics help to address the challenges of designing undergraduate projects in pure mathematics; they see applied math as naturally more accessible and less in need of this type of workshop. This issue will be discussed further at the REUF 2 reunion.

Changes made between REUF 1 and REUF 2, including providing background material, offering flexibility is software used, having more time for work in research groups (thus less group discussion), and follow-up activities, were all seen as improvements. The background reading and information provided was helpful. Either a list of topics found in standard undergraduate texts and a list of such texts, or a 10 page excerpt of a book, were felt to be sufficient. It made the participants more comfortable with projects outside their fields. Some projects did not require the use of computer programming. The Sage computer software was very useful for some projects, but the flexibility to use software that participants were already familiar with was an advantage for those projects that needed computing. Group discussions were informative, but most participants preferred to work on projects over group discussions, as many have very limited time for research at their home institutions.

The follow-up envisioned after REUF 1 was a reunion of participants to discuss research activities with undergraduates that resulted from the workshop, to be held at the JMM about 6 months after the workshop. The REUF 2 reunion will be held at the JMM in San Francisco. However, based on some recent outcomes from REU 2 and REU 1, we are exploring longer-term follow-up activities that focus on research rejuvenation for faculty (in addition to research activities of faculty with students).

Some REUF 2 groups expressed interested in an AIM SQuaRE type follow-up to continue their projects, meeting for a week once a year for two additional years. This would greatly facilitate the continuation of the project to the level needed for publication in an SCI research journal and for continued work by participants in a field that was new to them at the beginning of the REUF workshop.

One REU 1 participant spent a month visiting Leslie Hogben in conjunction with a 2009 REU program. During this time they established a research collaboration. The first joint paper is being prepared for submission. This visit also let the participant expand her research into linear algebra, an area to which she was introduced at REUF 1 and in which she plans to continue working.

Both of these activities would normally require additional external funding. These and additional types of follow-up activities will be explored at the REUF 2 reunion at JMM in 2010.

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