All roads to the KPZ universality class

organized by
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Workshop Summary

Overview

The Kardar-Parisi-Zhang (KPZ) universality class encompasses a broad range of probability models that typically describe random growth interfaces through height functions. These models share four key properties: local dynamics, smoothing mechanisms, slope-dependent growth rates, and space-time independent random forcing. The defining feature of the KPZ class is that seemingly different physical systems exhibit identical limiting statistical behaviors after appropriate rescaling.

The workshop was structured around the recent developments in KPZ universality. Let us recall the approximate KPZ timeline:

(1) **KPZ Equation Era (1986-2000)** began with the seminal paper by Kardar, Parisi, and Zhang introducing the stochastic partial differential equation (SPDE) that bears their names:

$$\frac{\partial h(x,t)}{\partial t} = \frac{\partial^2 h(x,t)}{\partial x^2} + \left(\frac{\partial h(x,t)}{\partial x}\right)^2 + \xi(x,t), \quad \text{where } \xi(x,t) \text{ is a space-time white noise.}$$

This non-linear SPDE describes the evolution of a height field $h(x,t)$ and predicts the characteristic $1:2:3$ scaling: height fluctuations scale as $t^{1/3}$, while spatial correlation length scales as $t^{2/3}$.

(2) **Integrability Era (2000-2020)** is driven by integrable probability methods, starting from the work of Johansson (2000) establishing the Tracy-Widom fluctuations of TASEP (Totally Asymmetric Simple Exclusion Process), and continuing through many contributions focused on case-by-case proofs that different models, including ASEP, random polymers, stochastic vertex models, and the KPZ equation itself share the same asymptotic statistics.

(3) **Universality Era (2020-present)** is characterized by the emergence of universal objects like the KPZ fixed point, Airy sheet, directed landscape, and stationary horizon that fully describe the entire universal asymptotic behavior.

The workshop brought together researchers who have contributed to these foundational developments or stand to immediately benefit from them. The workshop facilitated deep technical discussions and collaborations among the participants.

Perspectives

The workshop featured two morning talks each day, designed to balance comprehensive overviews with discussions of recent advances in the field. The presentations followed a natural progression in complexity and specificity throughout the week.

The initial talks centered on fundamental objects arising from the asymptotic analysis of KPZ models, including the KPZ fixed point and the stationary horizon. These presentations established a common theoretical foundation for all participants, regardless of their specific research focus within the broader framework.

On Wednesday, the talks shifted toward recent advances in the field. Speakers presented new analytical tools such as log-concavity techniques, large deviation estimates for polymers in general environments, deep integrability methods for stochastic vertex models, and methods to upgrade convergence from the KPZ fixed point to the full directed landscape. Several presentations explored fresh perspectives including connections between quantum models and KPZ asymptotics, as well as developments in the two-dimensional stochastic heat equation. Both the quantum systems and the two-dimensional directions were subsequently explored in the afternoon working groups.

Open problems

On Monday afternoon, we conducted a moderated problem session led by Evan Sorensen (moderator) and Hieu Trung Vu (scribe). This resulted in the identification of approximately ten significant open directions in the field, most of which were too early to formulate as concrete problems. Each subsequent afternoon, participants organized into working groups focused on addressing these challenges, with most problems receiving substantive attention.

The complete list of open problems identified during the workshop is available at https://lpetrov.cc/kpz2025/index.html. In the following sections, we discuss some of the key insights and partial results that emerged from these collaborative discussions during the workshop. Each of the open problems was assigned a *contact person* as a point of contact for any updates on the corresponding problem from any of the participants. In the following sections, each contact person wrote a short report on (1) a short summary of the problem and activity from the workshop, and (2) any updates and/or progress (i.e. the status) of the open problem.

Multilayer models and multilayer KPZ FP.

(1) The broad idea of the original problem was to explore whether it might be possible to extend methods used to get formulas for PNG with general initial condition to multilayer versions of PNG. But we realized relatively quickly that it is not at all clear what the good analog of multilayer PNG should be for general initial conditions, since part of the nice properties of multilayer PNG seem to be lost in the general case. The discussion then turned towards whether there may be a flat analog of the Airy line ensemble, an interesting problem which goes in the same broad direction. One possible candidate was put forward, but unfortunately we realized later on that it does not appear to have the right scaling to produce a line ensemble.
(2) No updates since the workshop, at least that I know of.
(3) Contact person: Daniel Remenik

Two-dimensional particle systems and continuous growth models.

(1) The problem concerns understanding the behaviors of growth models in the 2+1 dimensional KPZ universality class. The discussion focused on the possibility of extracting the fluctuation scaling exponent from the replica calculations, understanding

the literature on this direction, and trying to construct tractable 2+1 dimensional models from 1+1 dimensional models.

(2) Tomohiro Sasamoto and I had a brief and informal discussion. Besides this, I am not aware of any further development

(3) Contact person: Li-Cheng Tsai

Log-concavity.

(1) In the workshop we considered several questions concerning log-concavity of sequences arising in the context of growth processes of KPZ type. We mainly worked on two questions:

Q1 Log concavity of probability mass function for the longest increasing subsequence in a uniformly random permutation of n elements.

Q2 Log concavity of transition probabilities of the Totally Asymmetric Simple Exclusion Process with n particles.

During the workshop, the majority of the focus went into Question 1. This is an open problem of considerable interests in algebraic combinatorics. To attack it we considered a combinatorial construction reminiscent of the Gessel-Viennot involution of paths. Such construction allows to prove combinatorially the log concavity of the probability mass function for the longest increasing subsequence of a uniform random permutation, but with the important caveat that the permutation has random length with Poisson law. Since this approach just falls short from proving the desired result we have considered several possible modifications, with no conclusive results during the workshop.

We also briefly worked on Question 2, which appeared more promising, utilising analogous ideas to those discussed for Question 1.

(2) After the workshop, there have not been further updates.

(3) Contact person: Matteo Mucciconi.

KPZ behavior in quantum spin chains.

(1) This problem considers the limiting fluctuations of 1D quantum spin chains. There are some partial results indicating the precense of KPZ statistics in this setting. We discussed the know results in the literature and we discussed different approaches to attempt to extend the results to determine if KPZ statistics are universal for 1D quantum systems. In particular, we discussed discretization of the XXZ spin-1/2 chain via the Suzuki-Trotter decomposition which leads to a system with non-intersecting path with signed measure, resembling other models in KPZ. The main obstacle is the analysis of the signed measure, which is typically a positive measure for models in the KPZ universality class.

(2) No further discussion since the workshop.

(3) Contact person: Tomohiro Sasamoto.

Colored particles on the ring.

(1) In our discussions we made some preliminary observations about what the nature of a "periodic color-position symmetry" would have to look like, if it existed. From

our discussions, it seemed like there was some obstacle to constructing an analogous symmetry for the periodic colored model, namely that color-merging projections may not work in the correct way. Nevertheless, details would have to be checked, and other related questions are left to be explored. One group member, Kailun Chen, said he plans to work out details, but I am not aware of whether or not further progress has been made.

(2) Since the end of the conference, I have also searched for a construction of a periodic analog of the RSK map via the Yang-Baxter equation. However, thus far, it seems that any naive attempt to generalize what has been done on the line to the case of the cylinder does not work well, except for in the stationary situation. On the other hand, such a generalization has already appeared in the literature for certain "damped" periodic models. For the periodic KPZ equation, there is work that develops (in positive temperature) a certain periodic analog of the Pitman transform (which is essentially RSK), but its algebraic origins are unclear. Going forward, it remains interesting to clarify from an algebraic perspective what structures from the line admit generalizations to the periodic setting, and what exactly distinguishes these "damped" models from the usual ones.

(3) Contact person: Matthew Nicoletti

Longest common subsequence.

(1) The longest common subsequence (LCS) problem is a classic problem in computer science and combinatorics, which has applications in computational biology. Given a fixed alphabet, one can sample two independent words of equal length n, where the letters are sampled uniformly. Then the problem is to find the limiting distribution of the length of the LCS of these two words. Computer simulations suggest that the limiting distribution should be Gaussian, but there is still no mathematical proof of this conjecture.

During the workshop at AIM, we learned what is known about the limiting behavior of the length of the LCS. We reviewed several papers in which simulations were done and several related models were studied. We spent a good amount of time comparing the LCS problem with the last passage percolation model. We found out that the two models are described by the same discrete stochastic equations, with the only difference that the driving noises in the LPP model are independent and in the LCS problem they are dependent but uncorrelated.

(2) We keep thinking about the problem, communicating via email and Zoom calls. There are no significant findings to share at this moment.

(3) Contact person: Konstantin Matetski

KPZ line ensemble and random permutations.

(1) This problem was not discussed much at the workshop. I only learned at the workshop that there are two versions of the Gibbs property: one with, and one without, the k in the exponent in the interaction between levels k and k+1. Without k, the interaction is weak and the k-th path in the line ensemble may randomly grow large. This uniform Gibbs property is easier. On the other hand, the k-deformed Gibbs property makes the hypothetical permutation likely finitary. This problem (show that this

permutation is finitary; or if not, show that nothing will happens, like maybe the permutation has bounded displacement) might be the first accessible point of attack,
(2) No further updates at the moment.
(3) Contact person: Leo Petrov

Busemann functions and LPP.

(1) The problem is to understand the relationship between the RSK correspondence and the joint distribution of Busemann functions in last-passage percolation. Previously, a description of the joint distribution was given in terms of mappings of independent random walks. One can construct these independent random walks via the RSK correspondence by considering multi-path passage times.
(2) We made some good progress during the workshop. Since the workshop, we have had some discussions over e-mail.
(3) Contact person: Evan Sorensen

Relaxing/perturbing integrable models.

(1) The project is to consider probabilistic systems which would not be directly accessible by integrable techniques, but which can be coupled with multi-layer Gibbs measures and thus studied. At the workshop we discussed this problem in general terms, but we did not make any concrete progress or identify particular systems of interest.
(2) There has been no progress since the workshop.
(3) Zoe Himwhich

ASEP on graphs or trees.

(1) ASEP systems on trees or graphs—beyond the full line, half-line, open interval, or periodic lattice—form a large class of models that remain relatively less understood. Many natural and interesting questions arise for these models, including the characterization of invariant measures, mixing times, time-dependent behavior, current fluctuations, and large deviations.

    During the workshop, we devoted one afternoon to this problem. We identified two simple systems that are of interest:
    S1 TASEP on an (infinite) binary tree where only one node has two children, and
    S2 TASEP on the full line with a particle removal mechanism at site 0 (where particles disappear from the system at a fixed rate).

    We initially tried to solve these systems using the coordinate Bethe ansatz but did not pursue this direction further. We then focused on studying the invariant measure of the second system using the matrix product ansatz. On an open interval, the matrix ansatz for open ASEP relies on a "telescoping structure," as in the work "Exact solution of a 1D asymmetric exclusion model using a matrix formulation" by Derrida, Evans, Hakim, and Pasquier. A key challenge in applying the matrix ansatz to our system lies in identifying the appropriate telescoping structure.
(2) We agreed to continue working on this problem after the workshop. However, there has been no progress since then.
(3) Contact person: Zongrui Yang

Abbreviations

- USC: Upper semi-continuous function
- KPZ FP: KPZ Fixed Point
- ASEP: Asymmetric simple exclusion process
- PNG: Polynuclear growth
- KPZ: Kardar-Parisi-Zhang
- LPP: Last passage percolation
- FPP: First passage percolation
- GUE: Gaussian unitary ensemble
- XXZ: Anisotropic Heisenberg model
- LCS: Longest common subsequence
- RSK: Robinson-Schensted-Knuth
- GFF: Gaussian free field
- DL: Directed landscape

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