Sustainability problems

organized by
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Workshop Summary

This workshop brought together mathematicians, graduate students, and industry and public agency representatives to work on four problems of sustainability, including sustaining aquifers, power networks with renewable energy integration, energy harvesting using piezoelectric material, and the Tesla turbine in liquid-vapor fluid flow. On the first day, the problems were introduced by experts from Driscolls, EPRI, Boeing, and JPL. The participants worked in four groups on these problems with daily reports and interactive discussions.

Problem 1

The problem from Driscolls, also called the *Berries* problem during the workshop, was introduced and presented by Kelley Bell, Director of Community Environmental Initiatives, and Seth Edman, Water Projects Coordinator. The team included Dan Balbas, Kelley Bell, John Chrispell, Seth Edman, Kathleen Fowler, Stacy Howington, Lea Jenkins, Mark Minick, Carl Rupert, Ann Schwend, Tsvetanka Sendova, Daryl Springer.

The Pajaro Valley farming area of the Monterey Bay is one of the world's largest producers of berries and vegetables. The water supply to thousands of farmed acres is from an underground aquifer that is slowly being depleted. The group worked on two related problems. One was a virtual farm model to find an optimization tool for water use through crop rotation. The other one was a 2D Runoff model which simulated stream flow during a precipitation event. For the problem of the virtual farm model, two objectives were to (1) create a virtual farm tool that incorporates planting and harvesting schedules, cost and profit data, crop rotation, and fallowing to understand the impacts on profit and water use, and (2) create an optimization framework and objective function and constraint model to determine optimal farming strategies that maximize profit while meeting a sustainability requirement. The four-month-period virtual farm model, which includes allocating individual acres for each crop and tracking rotation, was implemented in MATLAB and verified with Driscoll's sustainability experts and farmer (Dan). Users can make planting decisions on a two month schedule based on profits, water usage, and the harvesting 'rules' for each crop. For the optimization, the profit and water usage model that was developed as part of the virtual farming tool was used in the context of a linear, constrained optimization problem. Over 200 equality and inequality constraints were developed to describe the planting, harvesting, and rotational properties of a model berry farm. The decision variables were the percentage of each crop planted over a 2 month period for 4 years, including strawberries, blackberries, blueberries, raspberries, and cover crops. Preliminary results, although not plausible, make sense in that strategies should include only raspberries and fallow land within the rotation. Raspberries use the least amount of water and yield the highest profit.

Another research group worked on a 2D Runoff model. Based on the model, a subwatershed analysis is carried out to estimate runoff and infiltration from rainfall events. Results of these simulations should not be used without additional work, but do show the potential to estimate local flow rates and water depths needed to design effective engineering solutions for infiltration. As a related issue, the floodwater capture from rivers were analyzed, providing a preliminary estimate of the maximum river flow rate that might be maintained while still capturing enough water to achieve a sustainable system. It lacks specifics and is intended only to encourage further conversation among the relevant parties about the alternatives.

Problem 2

The problem was presented by Dr. Robert Entriken from EPRI. The team included Shengyuan (Mike) Chen, Emilie Danna, Robert Entriken, Kory Hedman, Mark Iwen, Wei Kang, John Marriott, Anders Nottrott, George Yin, and Qing Zhang.

There is an international push to increase our use of clean, renewable electric energy. Much of the renewable energy requirements will come from future investments in wind energy and solar energy. These mandates pose new technical challenges to operating the high-voltage electric grid. Solar energy and wind energy are not controllable generation resources like traditional generation ones such as coal, natural gas, nuclear, and hydro. These resources are intermittent and can have fast and unpredictable output fluctuations. The team worked on two related problems: (1) the determination of storage capacity that is required to produce reliable output power; (2) the feedback control using battery storage to stabilize the output power.

One technique to limit the impact of these intermittent resources on the grid is to couple them with energy storage, i.e., a battery. For instance, when a cloud passes over a solar panel, the output of energy can drop by 70% very quickly. A battery can be used to smooth out this fast drop in electric energy to the grid such that the net output to the grid has a much slower rate of change. During the workshop, the required battery capacity was formulated as a Linear Programming (LP) problem. Its solution determines the optimal (minimal) size of a battery such that the net output to the grid does not fluctuate beyond levels that the grid can handle. The optimization is under a series of constraints. The inequality constraints on the rates of changes are referred to as the ramp rate constraints. The model is a scenario based problem with the scenarios reflecting various potential energy output levels from the solar panel. Equality constraints are included to represent the relationship between the storage output, the solar panel input, and the net output to the grid. Constraints are also included to ensure that the energy storage devices charge is always non-negative; the storage devices charge never exceeds its maximum; the net output from the storage device is zero over the cycle that is being modeled, be it a day, week, or month, etc. Some simulations were carried out to verify the feasibility of the LP model.

For the feedback control, it was formulated as a problem of optimal control constrained by inequalities and differential equations. If the power fluctuation forecast is available, a moving horizon optimal control scheme was investigated. A pseudo-spectral optimal control method was applied to the model. From the simulations for a typical scenario, it is shown that the method is numerically stable. In some cases, control fluctuation may occur. However the problem can be easily solved by using adequate number of nodes in the numerical computation. If the system has significant unpredictable and/or random uncertainties, a state feedback control scheme was proposed. The control law and the associate Hamilton-Jacobi-Bellmans PDE, or HJB equation, was derived. A problem of future work is to solve, either analytically or numerically, the HJB equation for state feedback control.

Problem 3

The problem from the Boeing Company was presented by Dr. Thomas Grandine and the team included him, Stefan Llewellyn Smith, John Marriott, Hong Zhou, Michael Gratton and Ellis Cumberbatch.

The use of piezoelectric devices as sensors or current sources in inaccessible places has been researched, and the models described in published literature were the subject of our focus, with the intent of

(1) placing them in a coherent structure based on exact physics, and
(2) improving the mathematical approaches so far used.

The first task was accomplished quickly there are comprehensive and general descriptions available. The second task was accomplished in some specific cases. For instance there are a number of recent publications on the use of an oscillating beam on which is pasted piezo-electric patches. The beam bending creates voltages in the patches that can be harvested for external use. There are also models that take the current from one patch to excite enhanced vibrations further along the beam, thereby enlarging the current there. This aspect has been modeled as an optimization problem, but this area was not discussed by the team.

In the published literature the PDE that describes the motion of the beam contains delta functions arising from the voltages that exist only on the patches. The mathematical approach commonly used is to represent the solution in terms of time and spatial modes that are smooth. This approach has obvious convergence difficulties in approximating the discontinuities. The team was able to formulate an alternative approach that circumvents this problem. The published work is based on the approximations inherent in beam models that rely on small displacements. More exact **non-linear** formulations were discussed.

Problem 4

The Tesla Turbine problem was presented by Dr. Juan Cepeda-Rizo of JPL. The team that worked on this problem included Ali Nadim, Joseph Fehribach, Xiaoming Wang and Matthew Mata, with occasional help from Michael Gratton and Stefan Llewellyn Smith. While the main problem of interest to JPL is the flow of wet steam (i.e., vapor-liquid flow) through the turbine, a review of the literature indicated that little systematic mathematical modeling work on single-phase flow in the Tesla turbine has been performed and no basic result exists that predicts the angular velocity of the turbine as a function of the input pressure and the applied load, from which power output and efficiency can be inferred. Therefore, the team set out to examine single-fluid flow through the turbine first. Since compressibility of the gas powering turbines is known to be important, the first modeling attempts where with compressible flow equations. The gas was assumed to be isentropic so that the pressure and density could be related by a simple nonlinear equation of state involving the polytropic index. This was supplemented by the equations of conservation of mass and momentum, assuming axisymmetry, in the region between two rotating disks in the Tesla turbine. The viscosity-dominated regime could be solved completely, but provided no torque at leading order. Extending the calculations to small but finite Reynolds number would provide an expression for the torque at first order in Reynolds number. However, since the operating regime involves high Reynolds numbers, this model was not pursued and we instead focussed on the inertia-dominated regime. In the latter case, for high enough driving pressures, it was predicted that the flow becomes "choked" and a standing circular shock wave must form at some finite radius within the disks. Modeling this regime further can be fruitful and provide new mathematical results. At lower driving pressures, the base Euler flow could also be calculated without shock waves. To get the resulting torque and angular velocity of the disks, the effects of viscous boundary layers on the disks would need to be accounted for. Our team did consider the boundary layer problem; however, only for the incompressible limit where the core flow was obtained by solving the constant-density Euler equations. At that point, by assuming that the boundary layers are thin and uniform, we were able to derive an expression for the torque with or without applied loads. To investigate the boundary layer structure in more detail, the team also applied the Karman momentum integral approach to the 2D flow problem and upon making an assumption regarding the self-similar structure of the velocity profiles, was able to derive a second analytical result for the torque as a function of the applied load. Several of the approaches pursued by the team during the week are worthy of further research. Once single phase flow in the Tesla turbine is well understood, the case of vapor-liquid flow can be reconsidered.