

GENERALIZING THETA CORRESPONDENCES

organized by
Wee Teck Gan and Gordan Savin

Workshop Summary

The workshop followed the format of two talks in the morning and 2 -3 work sessions in the afternoon. The talks were:

- Monday: background on minimal representations and backward lifting by Savin and Hundley.
- Tuesday: classical real and p -adic theta correspondences by Trapa and Zorn.
- Wednesday: on Ichino-Ikeda formalism by Ichino and Lapid.
- Thursday: on the correspondences arising from the work of Loke and Savin by Loke and on Gross-Prasad conjecture for Arthur packets by Gurevich.
- Friday: on Gross-Prasad conjecture by Takloo-Bighash and on the second term identity for the Siegel-Weil formula by Takeda.

Afternoon activities were:

- Monday: we had two background building sessions. One was on wave-front sets for p -adic representations (Savin) and the other was on the Gross-Prasad conjecture (Takloo-Bighash).
- Tuesday: two working sessions. One was on the exceptional theta lift from G_2 to C_3 and its backward version. The other was on classical theta correspondences.
- Wednesday: the two working sessions from Tuesday were continued and we also had another background building session, on Shahidis L -functions and relation to reducibility of parabolic induction conducted by Goldberg.
- Thursday: two working sessions. One on extending a case of Ichino-Ikeda formalism to G_2 . The other was on comparing the classical theta correspondence to correspondences described in Lokes talk and their relevance to conjectures of Ibukiyama.
- Friday: a special talk by Weissman on E_8 and a summary of discussions.

Overall, the workshop went pretty well. Participants seem to have liked the format of the workshop. I can think of two principal reasons for this: (1) The format allows for more interaction than what happens in the usual conference. (2) Afternoon sessions are tailored according to need and interest, and they relate to topics covered in morning talks. We had a good group of graduate students and postdocs participating. The main weakness of this particular workshop was a lack of more senior people, due to a couple of last minute cancelations. As far as my work is concerned I have accomplished my main goals. More precisely:

- (1) We have been able to clarify relevance of Ibukiyamas conjectures to my work with Loke. According to Ibukiyama there should be a correspondence between half-integral weight modular forms of level one for \widetilde{Sp}_4 , and integral weight modular forms on $SO(3, 2)$. (In essence this is a genus 2 version of Shimuras lift.) One would expect

that this sort of question may be attacked by the classical dual pair correspondence but here is a glitch. Discrete series representations of $SO(3, 2)$ lift to generic representations of \widetilde{Sp}_4 . Thus in order to obtain holomorphic lifts on \widetilde{Sp}_4 we are forced to lift from a non-split form of $SO(3, 2)$. Such group, however, is necessarily ramified at some primes, so the lift will never produce a level one modular form! The work with Loke provides an alternative approach which does not create unwanted ramifications. We have checked that the correspondence of Loke and Savin is compatible with Ibukiyamas conjectures. It remains to work out the global and local (p -adic) lifts.

- (2) In a work session that took 2-3 days, working with Hundley, Lapid and Weissman we made a good progress on injectivity of the exceptional theta correspondence from G_2 to C_3 . (These two groups form a dual pair in E_7 .) We wanted to exploit methods of backward lifting coming from classical groups. The backward lift in the case of classical groups is usually hard to compute, but we figured out an easy way to do it in the case at hand, using the minimal representation of E_8 . Marty Weissman is currently writing up results. In the end, we should be able to reduce the Langlands classification for generic representations of G_2 to classifications for A_2 and C_3 .

AIM is a low key, yet very pleasant place to do mathematics. This is in large part due to AIMs efficient and professional staff. I am sure that most participants would agree with this assessment. Thank you!